



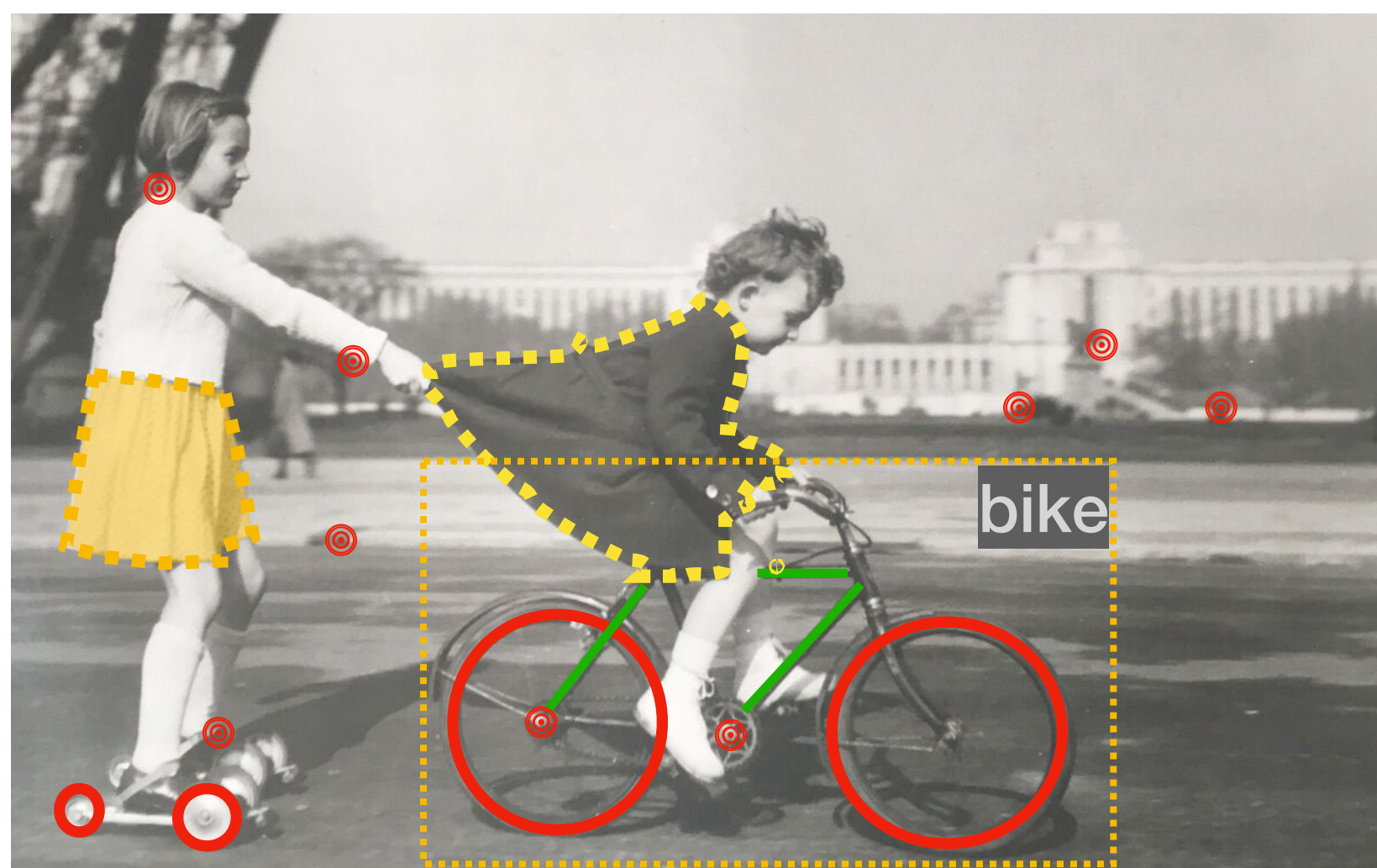
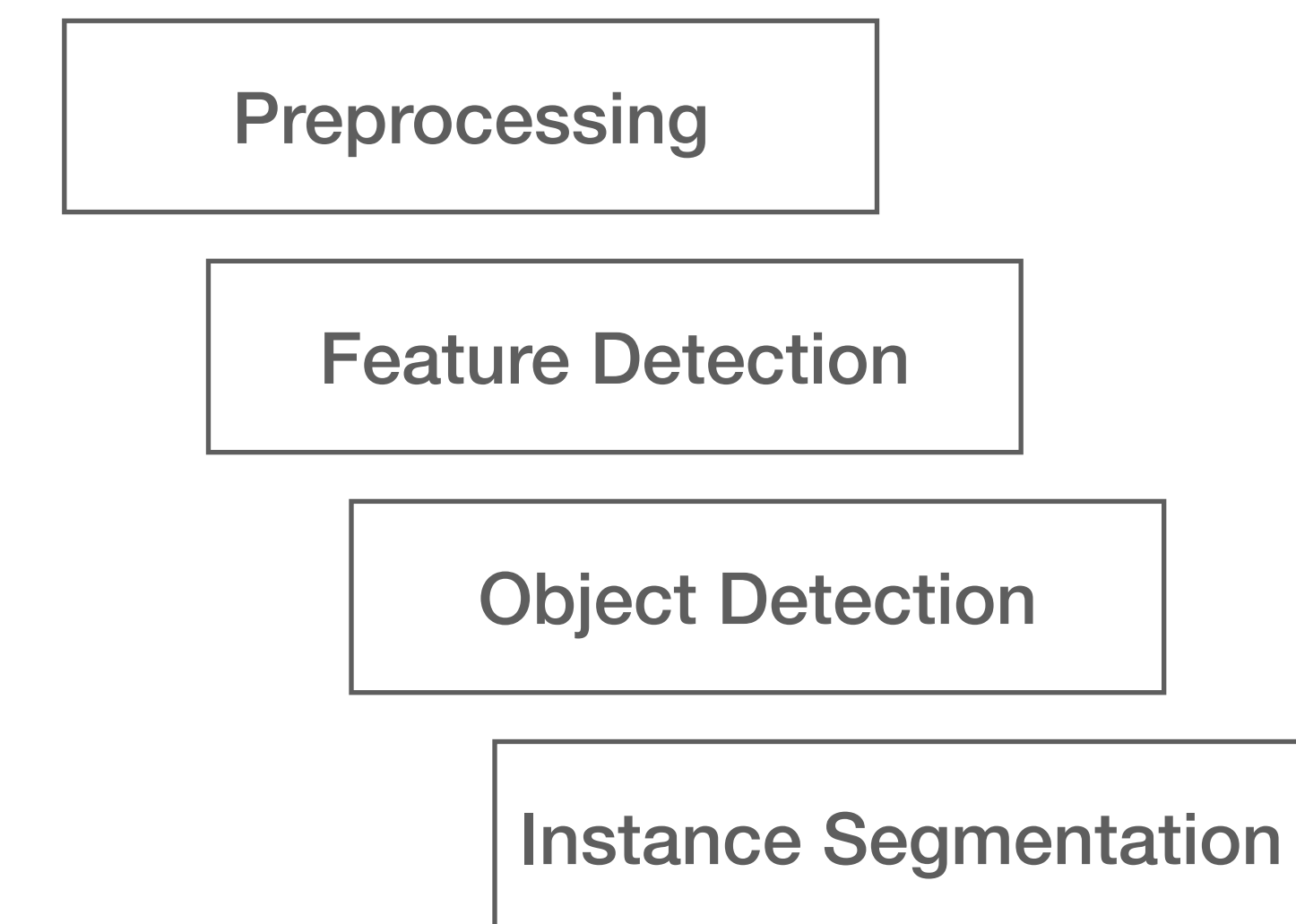
Bioimage Informatics

Daniel Sage & Arne Seitz

Course

Feature Detection

👁 Feature Detection / Object Detection



Visual Feature Detection

Multidimensional 2D / 3D

- ▶ Spot or blob detection ➡ DoG / LoG
- ▶ Edge detection ➡ Gradient
- ▶ Ridge detection (Filament) ➡ Hessian
- ▶ Segment ➡ Pixel Tracing
- ▶ Keypoints ➡ Gradient
- ▶ Line/Circle ➡ Hough Transform
- ▶ Simple parametric shape ➡ Fitting
- ▶ Texture Analysis ➡ Filterbank and classification
- ▶ Directional Image Analysis ➡ Gradient Structure Tensor

In this Course
2D





Gaussian Filter Definition

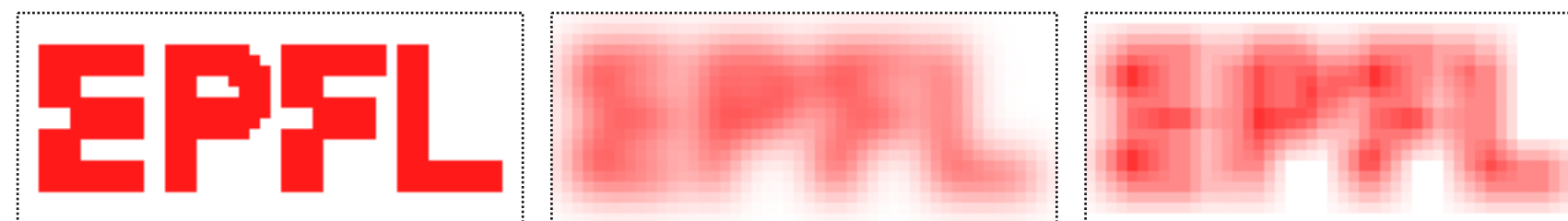
Multidimensional Gaussian Function

$$g_{\sigma}(\mathbf{x}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

↑
standard
deviation same
unit than x

- **Adjustable** in every dimension

- **Isotropic**
radial symmetry



- **Separability**

$$g_{\sigma}(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} \right)$$

- **Cascade**

$$\sigma_{eq}^2 = \sigma_1^2 + \sigma_2^2 \quad g_{\sigma_1}(g_{\sigma_2}(x)) = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{x^2}{2\sigma_1^2 + 2\sigma_2^2}}$$

e.g. shift t_0 in time, center $\mathbf{c}$ in space
 σ_s in space and scale σ_t in time

$$g_{\sigma}(x, y, t) = \frac{1}{(2\pi)^{3/2} \sigma_s^2 \sigma_t} e^{-\frac{(x-c_x)^2 + (y-c_y)^2}{2\sigma_s^2} - \frac{(t-t_0)^2}{2\sigma_t^2}}$$

Discretization of the Gaussian function

Truncated

0.059	0.097	0.059
0.097	0.159	0.097
0.059	0.097	0.059

$\sigma = 1$

0.000	0.000	0.001	0.002	0.001	0.000	0.000
0.000	0.003	0.013	0.022	0.013	0.003	0.000
0.001	0.013	0.059	0.097	0.059	0.013	0.001
0.002	0.022	0.097	0.159	0.097	0.022	0.002
0.001	0.013	0.059	0.097	0.059	0.013	0.001
0.000	0.003	0.013	0.022	0.013	0.003	0.000
0.000	0.000	0.001	0.002	0.001	0.000	0.000

$\sigma = 1$

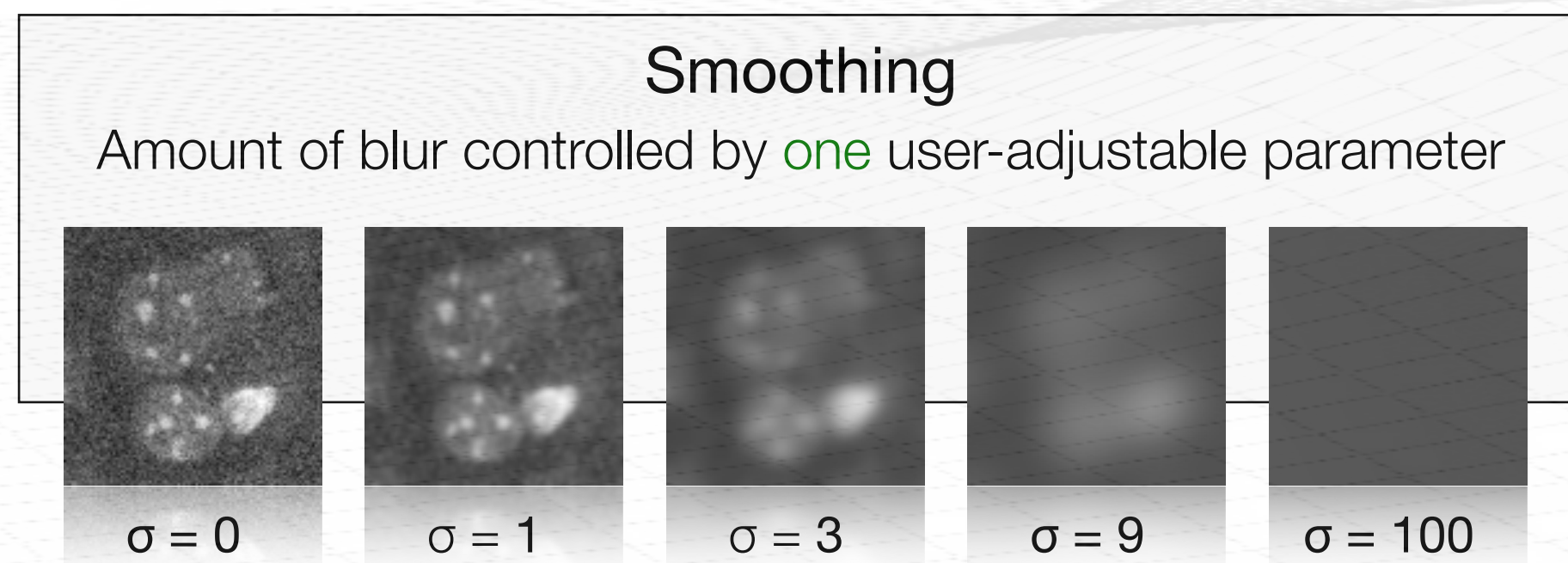
Support

$$N = 2 \lceil 3 \sigma \rceil + 1$$



Gaussian Filter Properties

- **Convolution** linear filter
- **Fourier** low-pass filter
- **Limit** converge to identity $g_0(x) = \delta(x)$
- **Normalization** unitary gain, no overshoot
- **Central limit theorem**
“any repetitive operator goes in the limit to a Gaussian”
FAST! independent of the filter size
(e. g. cascade of 3 exponential filters)

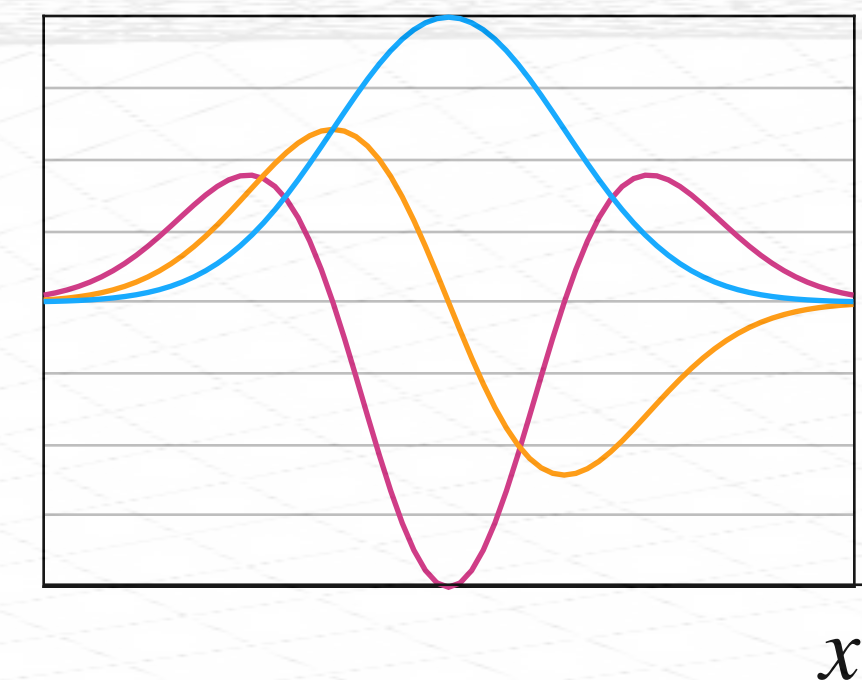


$$g(x) = e^{-\frac{x^2}{2\sigma^2}} \xrightarrow{\mathcal{F}} \hat{g}_\sigma(\omega) = e^{-2\pi^2\sigma^2\omega^2}$$

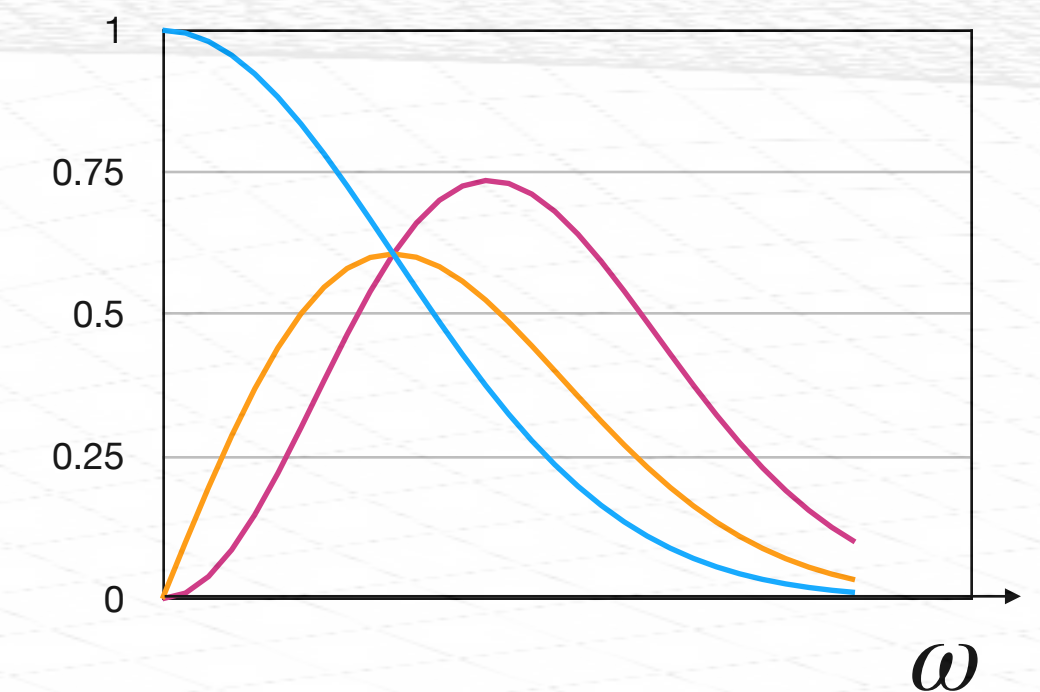
$$\frac{\partial g(x)}{\partial x} = \frac{-x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \xrightarrow{\mathcal{F}} \omega \hat{g}(\omega)$$

$$\frac{\partial^2 g(x)}{\partial^2 x} = \frac{x^2 - \sigma^2}{\sigma^4} e^{-\frac{x^2}{2\sigma^2}} \xrightarrow{\mathcal{F}} \omega^2 \hat{g}(\omega)$$

— Gaussian
— First derivative
— Second derivative



— Gaussian
— First derivative
— Second derivative



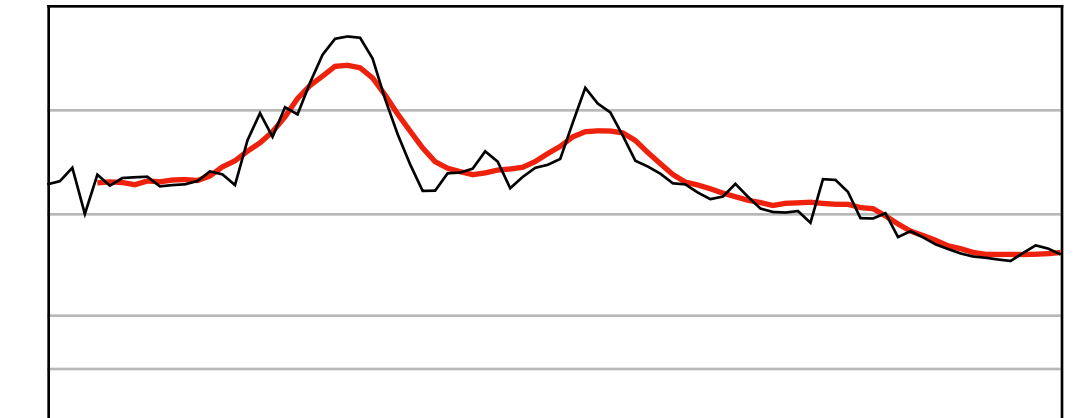
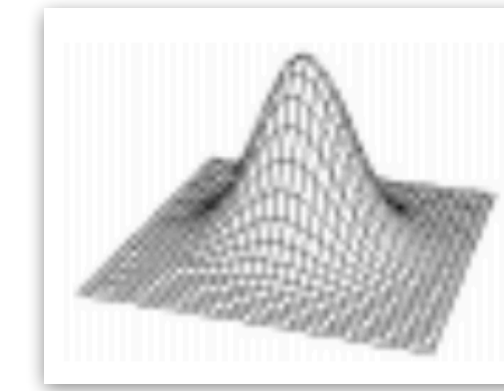


Gaussian Filter in Action

Gaussian

- Low-pass filter
- Denoising

$$g(\mathbf{x}) = e^{-\frac{\mathbf{x}^2}{2\sigma^2}}$$

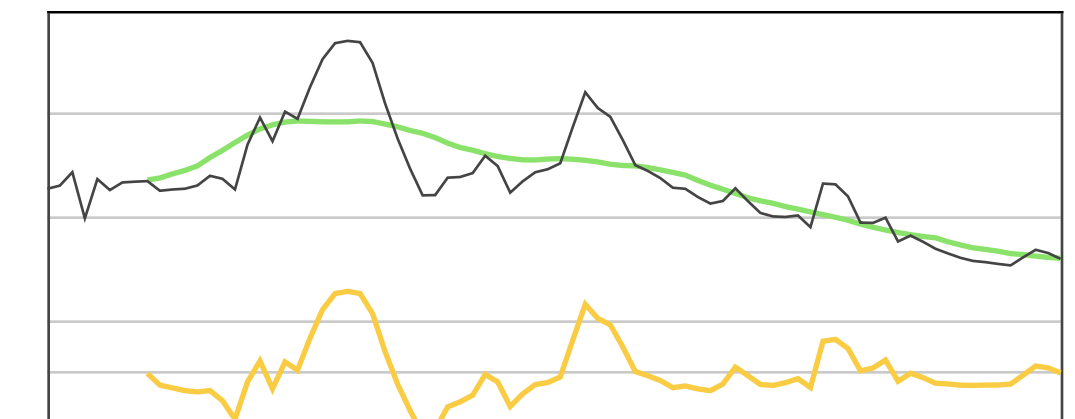


Signal - Gaussian

- High-pass filter
- Flat background

$$h(\mathbf{x}) = 1 - e^{-\frac{\mathbf{x}^2}{2\sigma^2}}$$

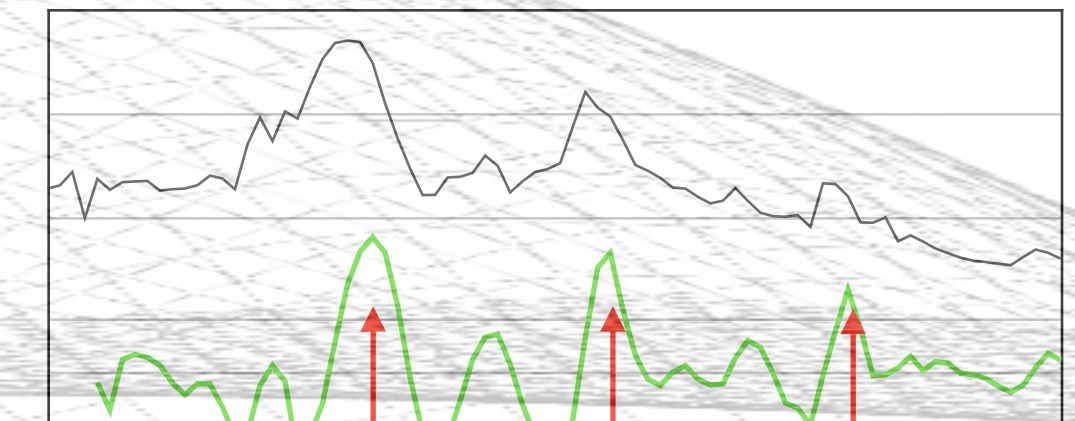
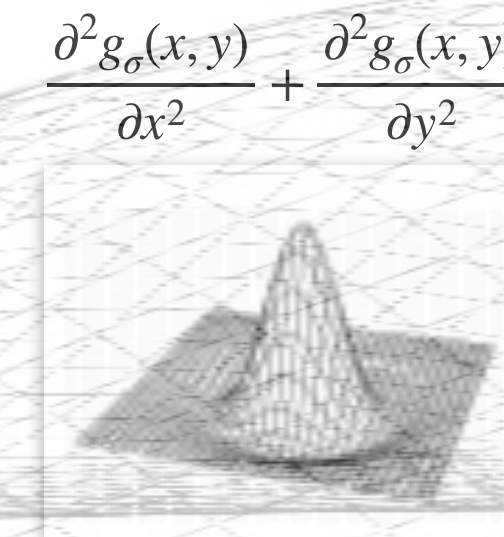
— Signal
— Large Gaussian filter
— Signal - Gaussian



Laplacian of Gaussian (LoG)

- Band-pass filter
- Spot detection: Local extrema

$$\Delta g(\mathbf{x}) = \frac{\mathbf{x}^2 - \sigma^2}{\sigma^4} e^{-\frac{\mathbf{x}^2}{2\sigma^2}}$$

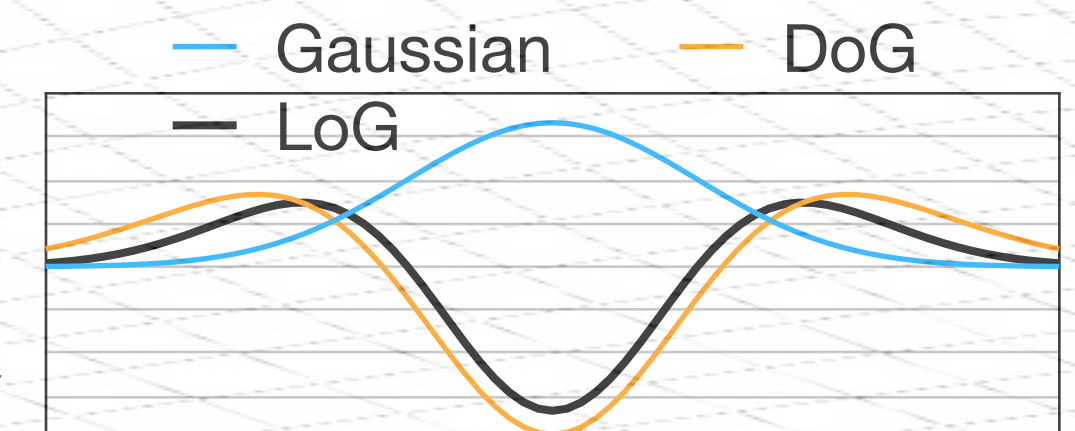


Difference of Gaussian (DoG)

- Band-pass filter
- Spot detection: Local extrema

$$\text{dog}_\sigma(\mathbf{x}) = \frac{1}{2\pi\sigma_1^2} e^{-\frac{\mathbf{x}^2}{2\sigma_1^2}} - \frac{1}{2\pi\sigma_2^2} e^{-\frac{\mathbf{x}^2}{2\sigma_2^2}}$$

$$\sigma_1^2 = 2\sigma_2^2$$



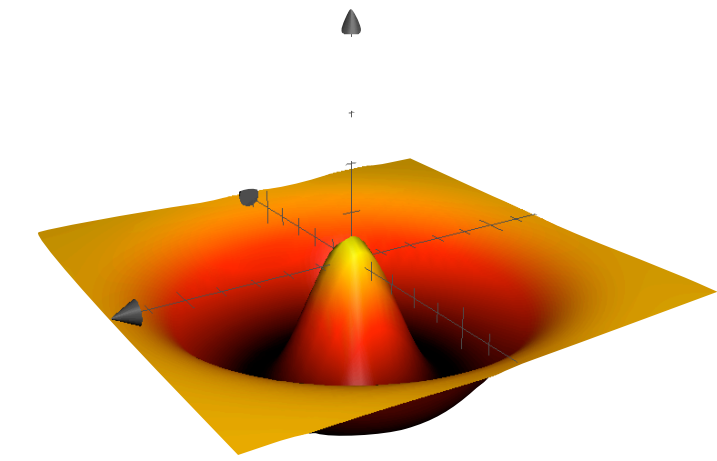


Spot Detection

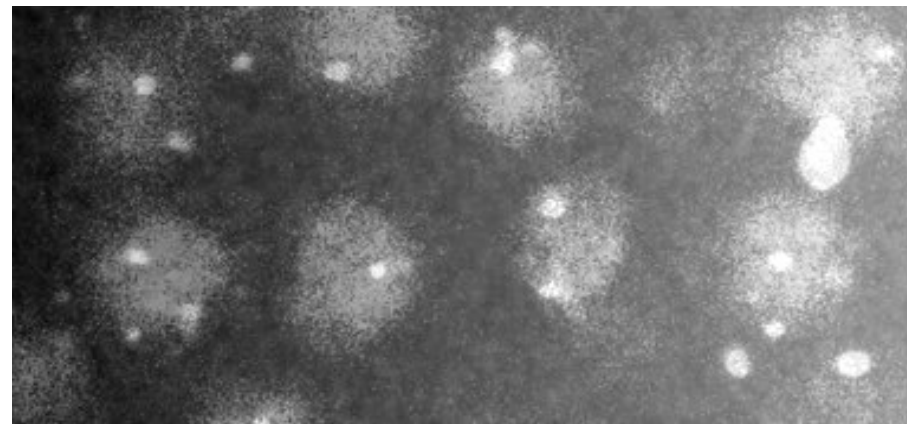
Local extrema on the output of the LoG or DoG

- Isotropic
- Bright spot (positive) or dark spot (negative)
- Filter 2D / 3D, tune the size $\sigma_x \sigma_y \sigma_z$

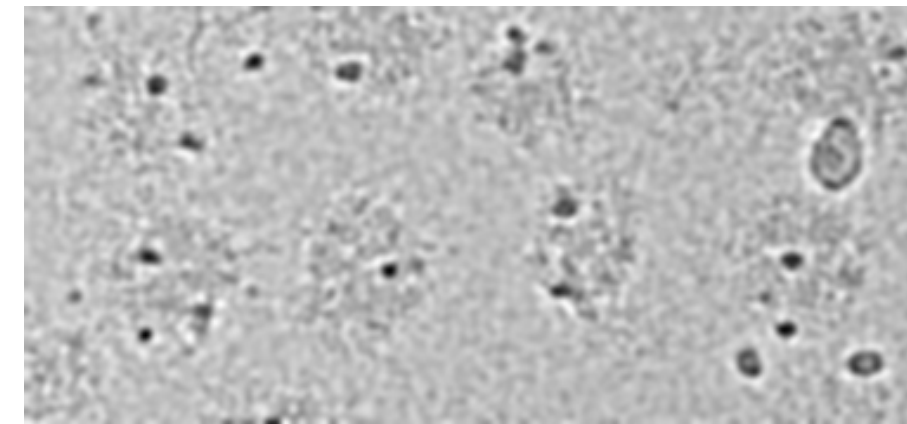
Mexican hat



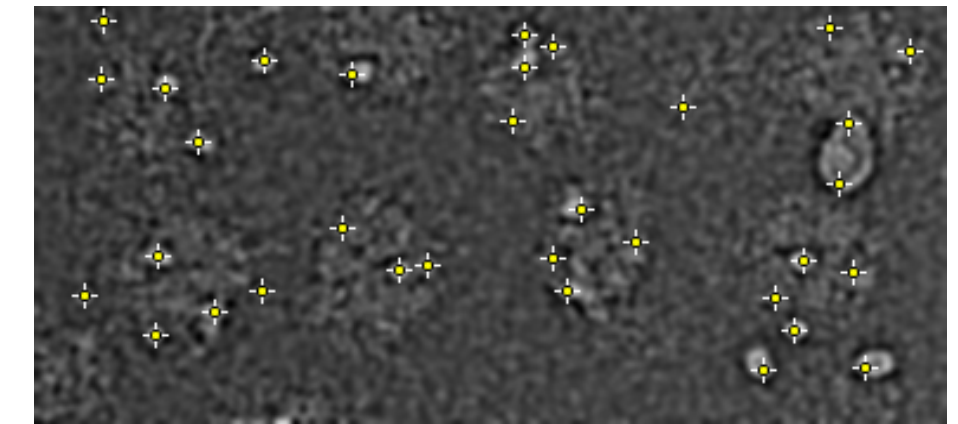
2D



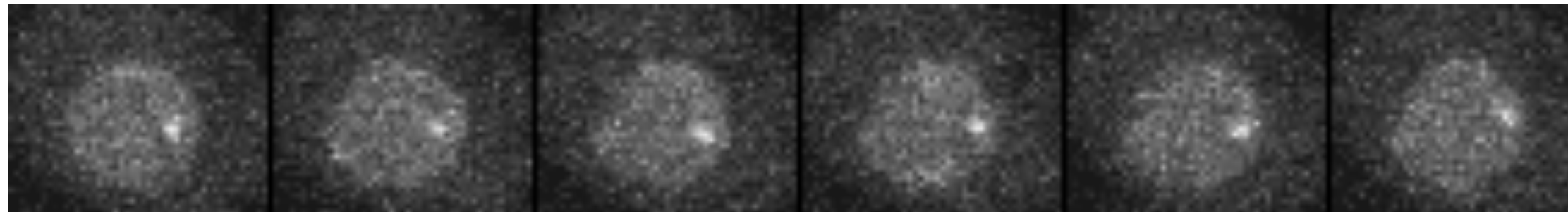
convolve



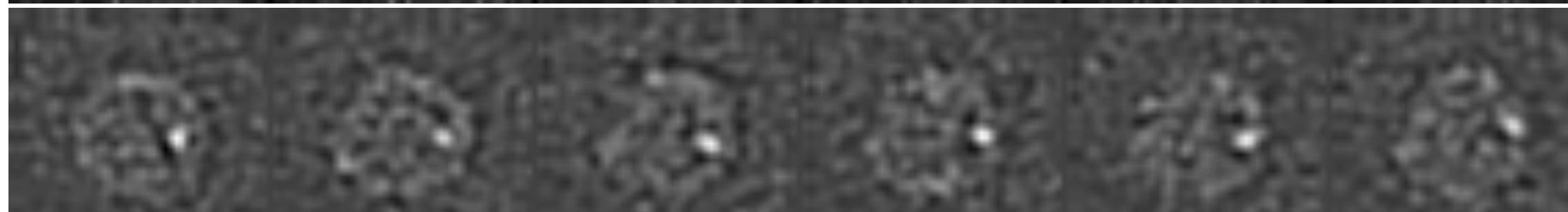
Local
min.
3x3



3D+T



input
MIP in z



$\sigma = 1.25$
MIP in z



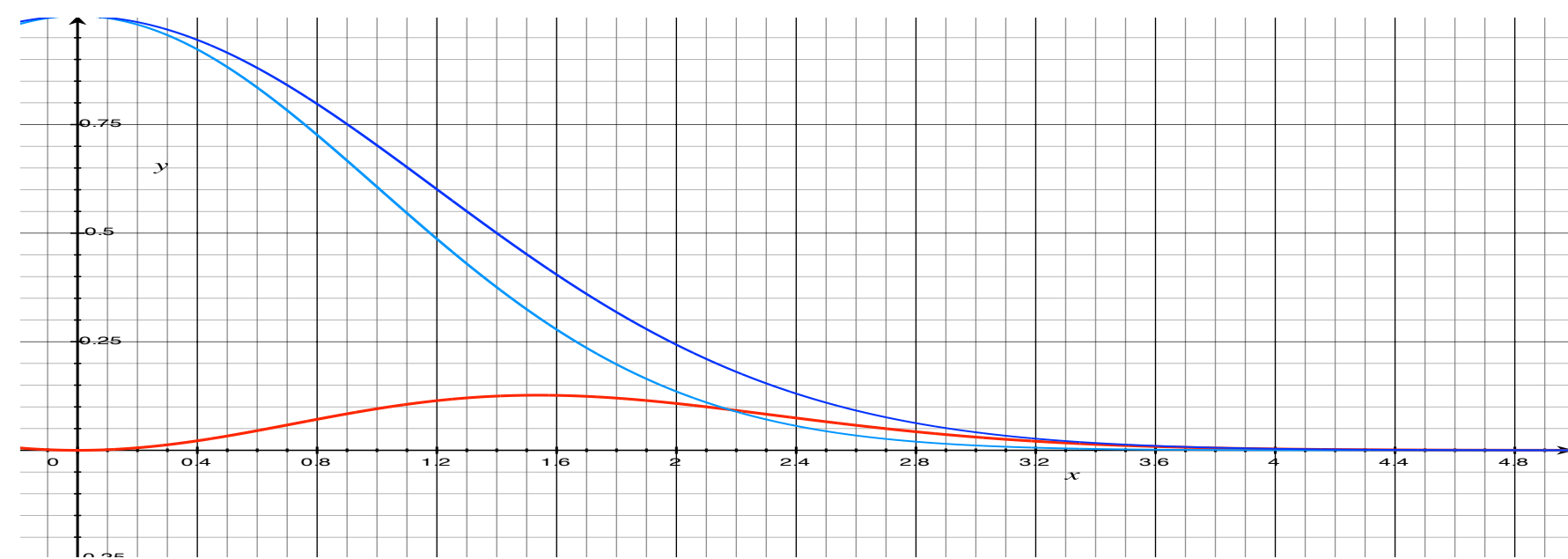
$\sigma = 0.75$
MIP in z



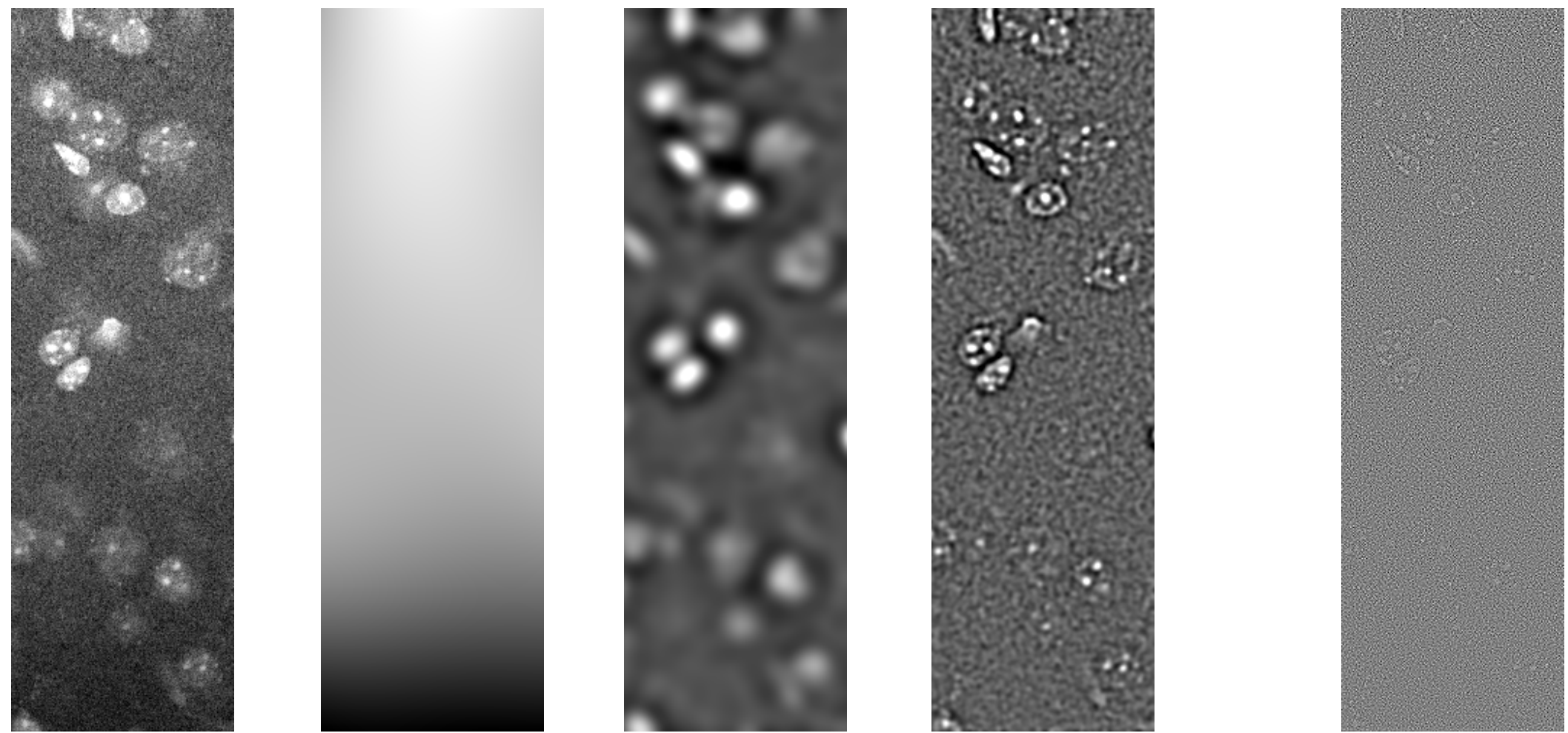
Blob Detection

Scale selectivity

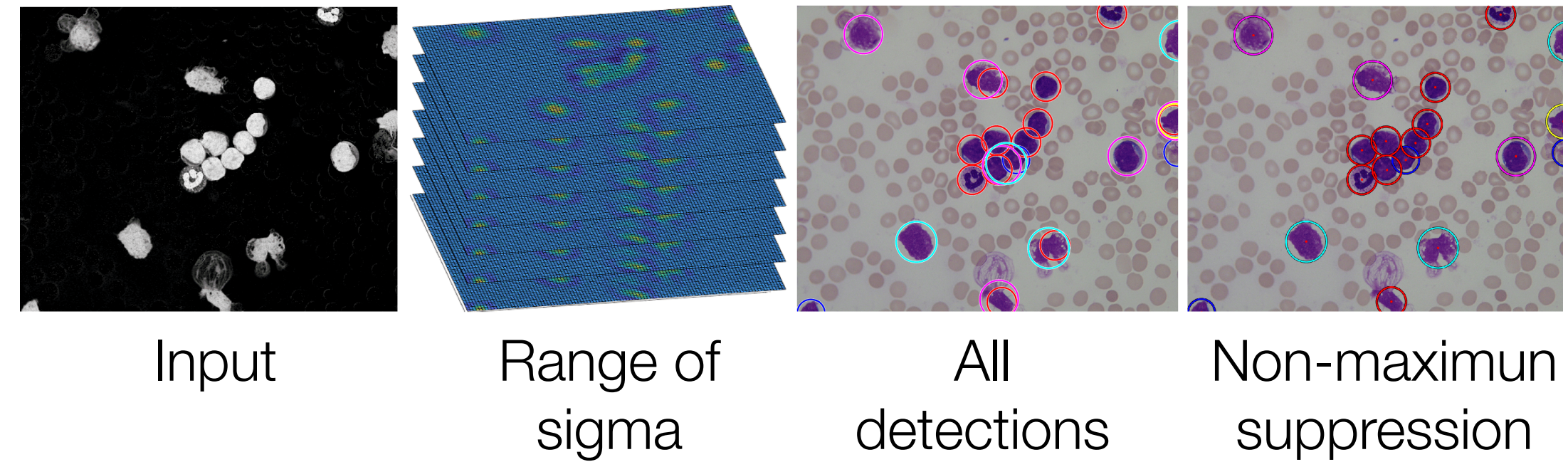
DoG
LoG



Background Nucleus Particles Noise



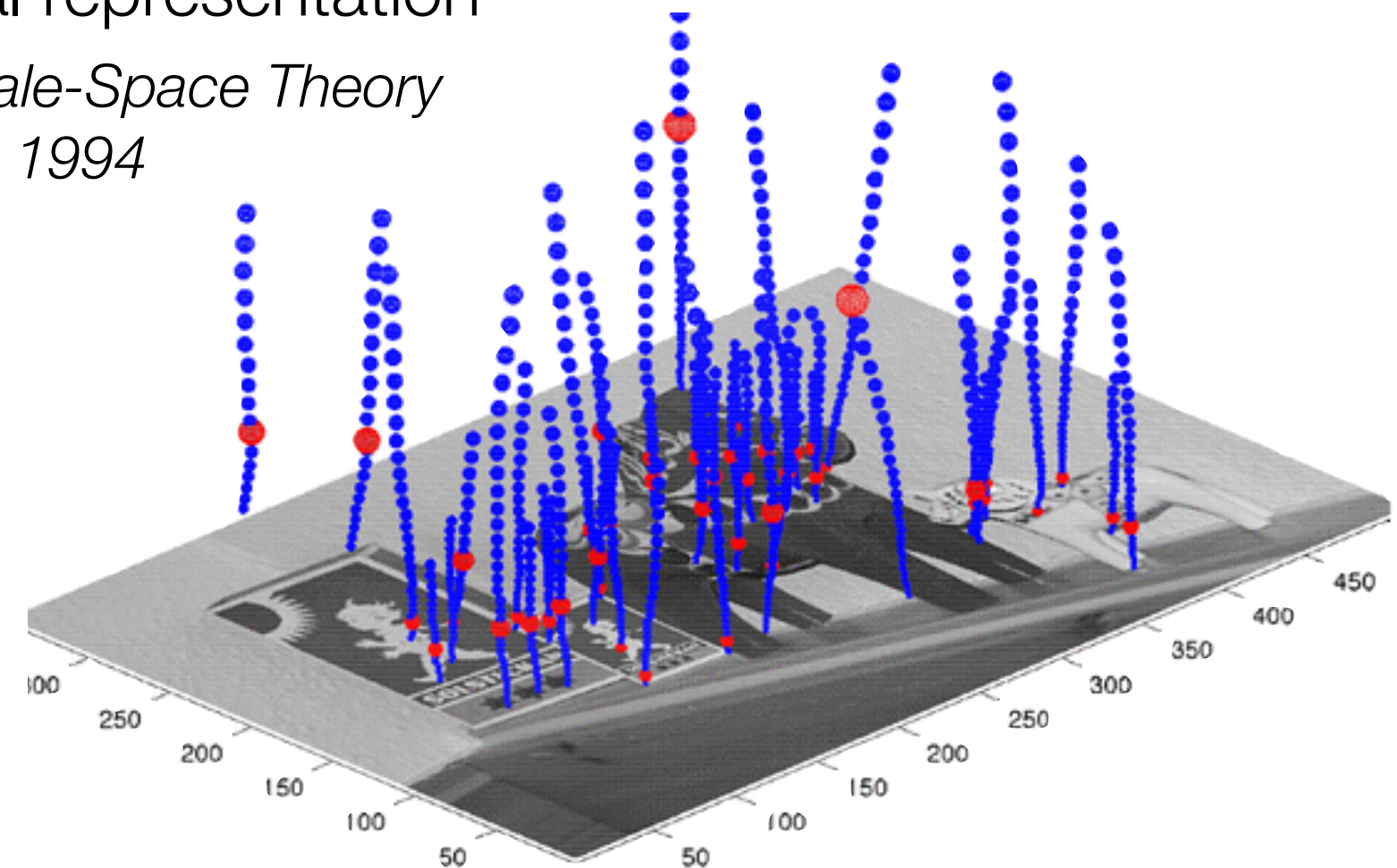
Multiple scale detection



Scale-space theory

Multi-scale signal representation

Tony Lindeberg, Scale-Space Theory in Computer Vision, 1994



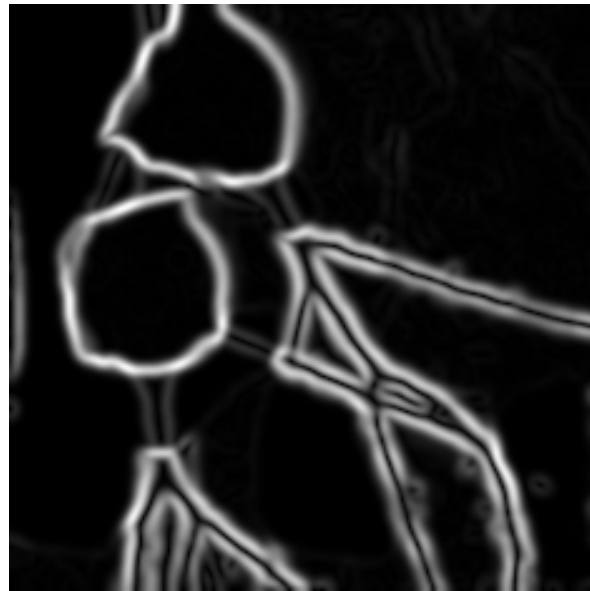
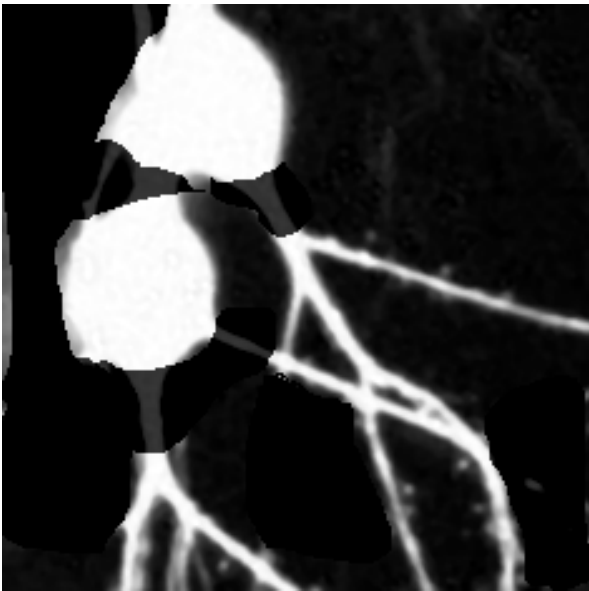


Multivariate Derivative

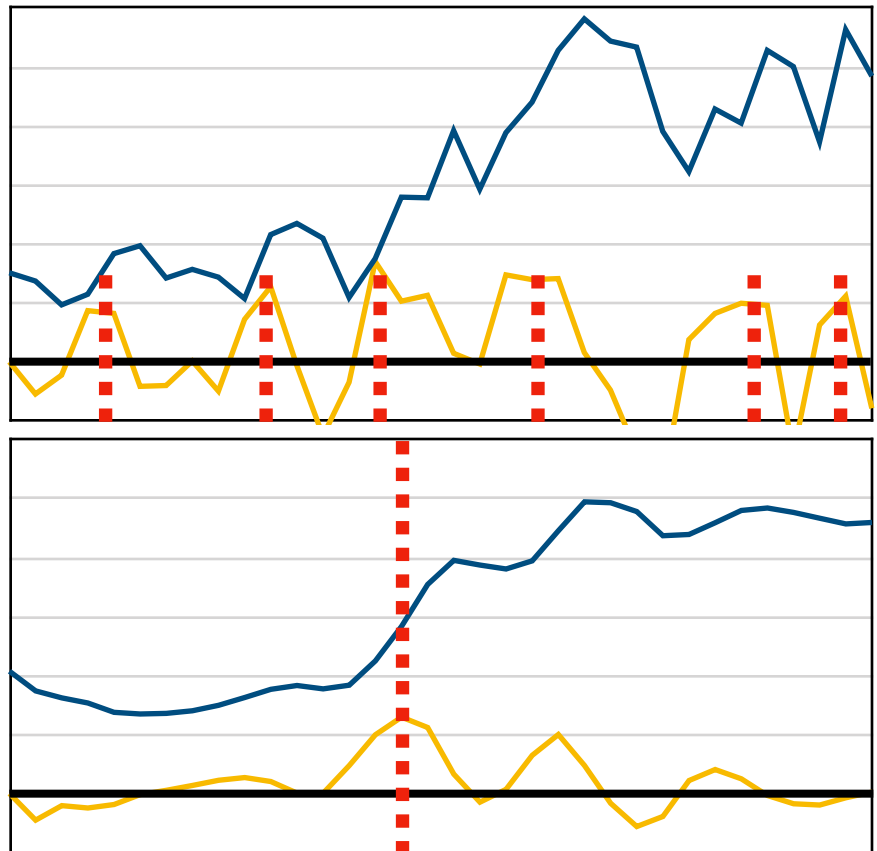
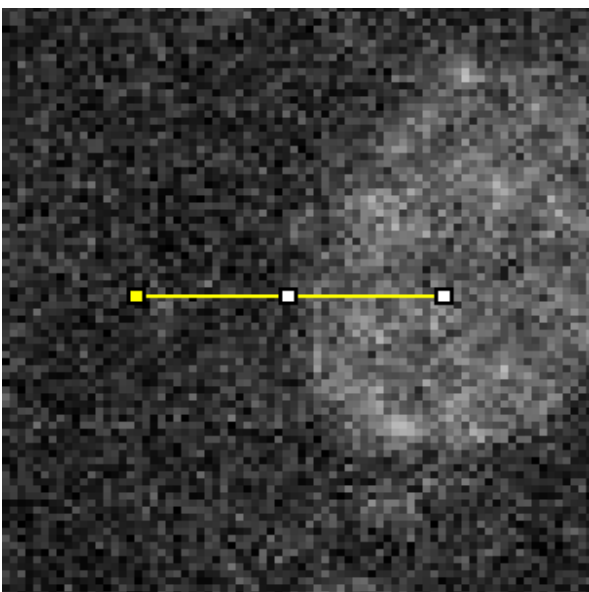
Gradient in 2D

Edges

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad |\nabla f| = \sqrt{\frac{\partial f^2}{\partial x} + \frac{\partial f^2}{\partial y}}$$



$|\nabla f|$



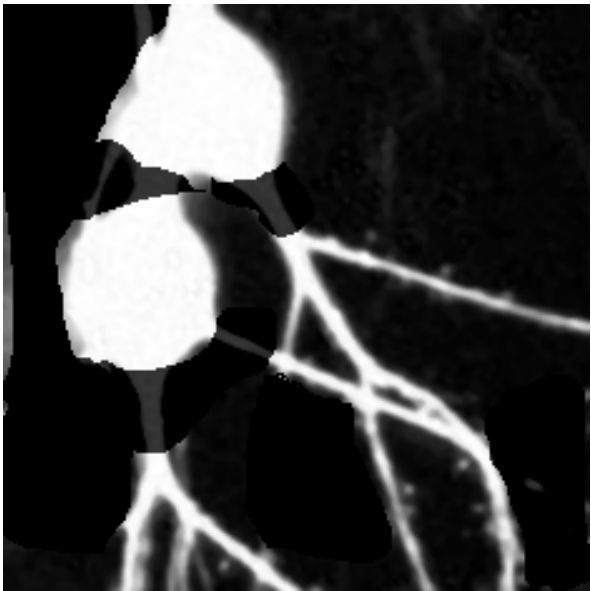
Localization of edges by the first derivative

Extrema on the first derivative of denoised signal

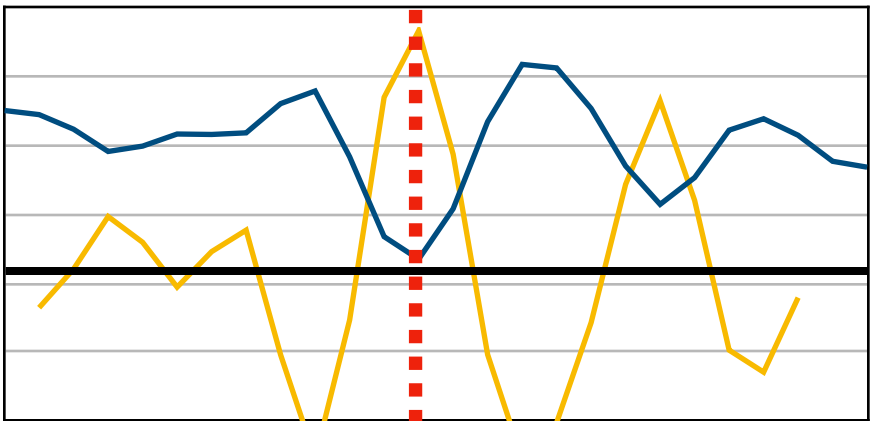
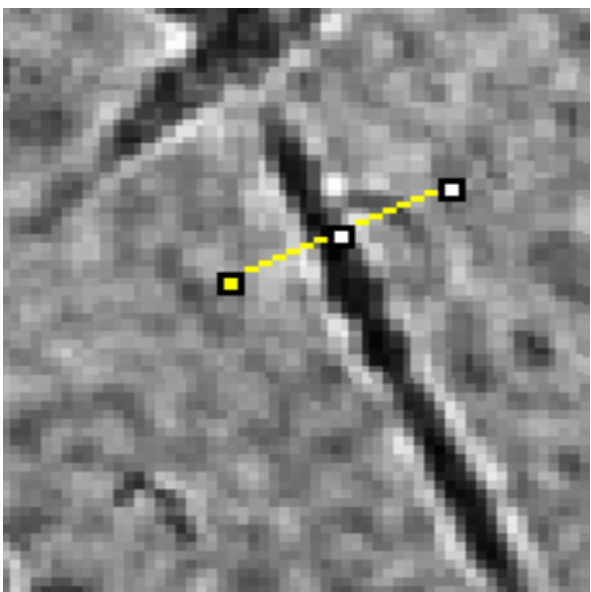
Hessian in 2D

Ridges

$$Hf(x, y) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} \quad \begin{bmatrix} \lambda_{\max} & 0 \\ 0 & \lambda_{\min} \end{bmatrix}$$



$\lambda_{\max}$



Localization of ridges by second derivative



Orientation of Edges / Ridges

Gradient in 2D

Edges

Hessian in 2D

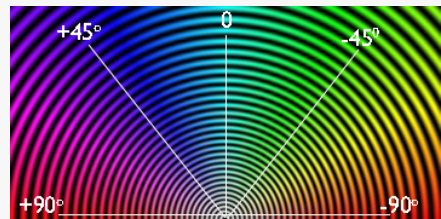
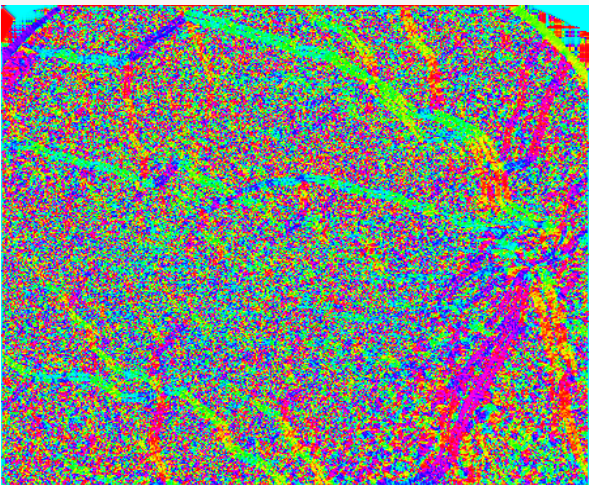
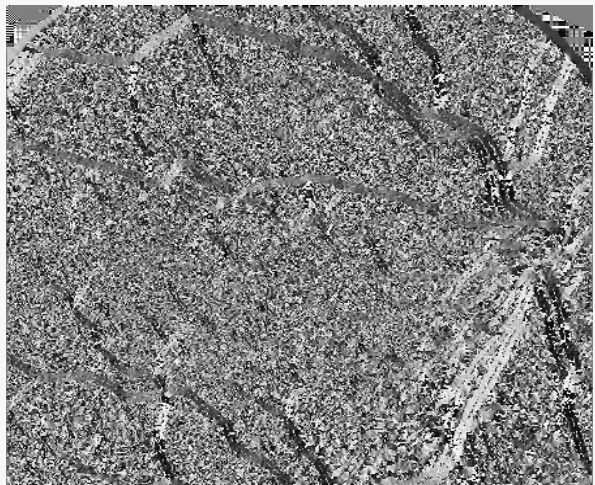
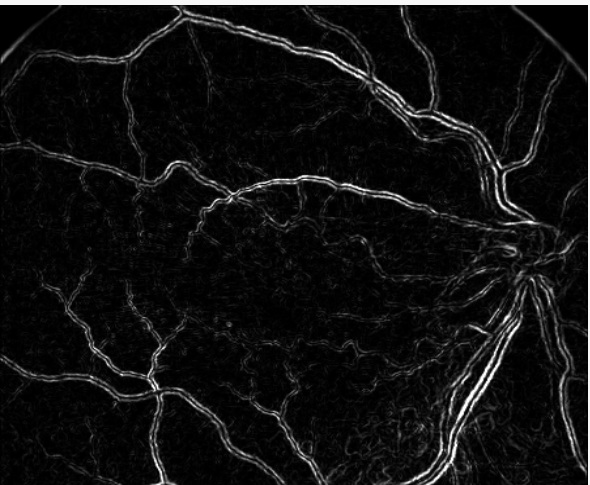
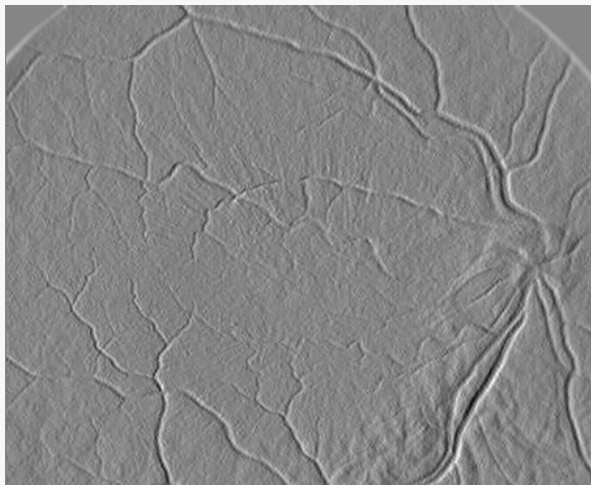
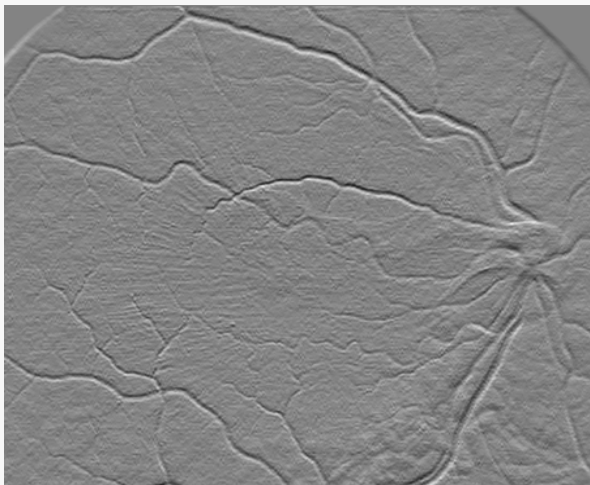
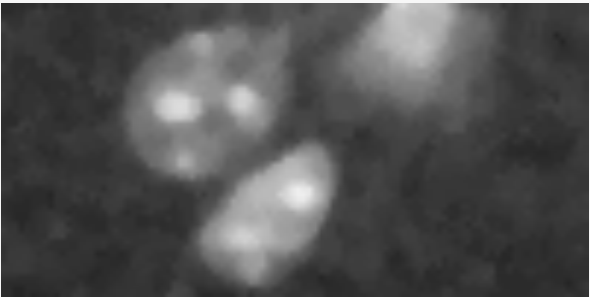
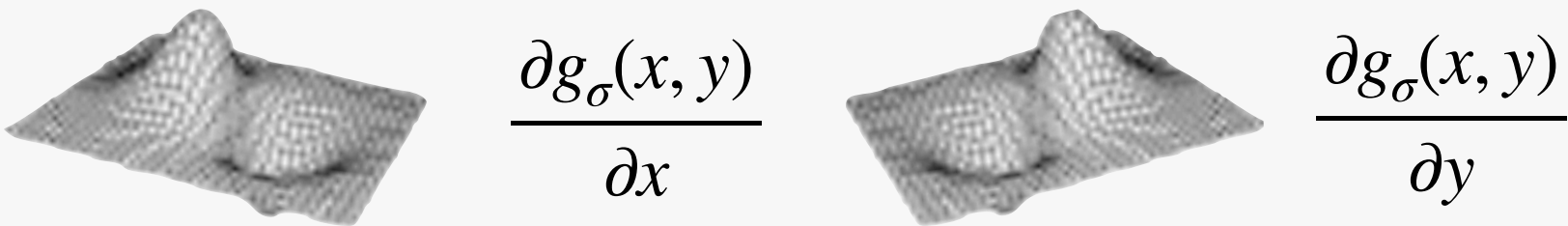
Ridges

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad \theta = \arctan\left(\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}\right)$$

$$Hf(x, y) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} \quad \begin{bmatrix} \lambda_{\max} & 0 \\ 0 & \lambda_{\min} \end{bmatrix}$$

$V_{\max}$

Direction of gradient with derivative Gaussian Filters



f_x

f_y

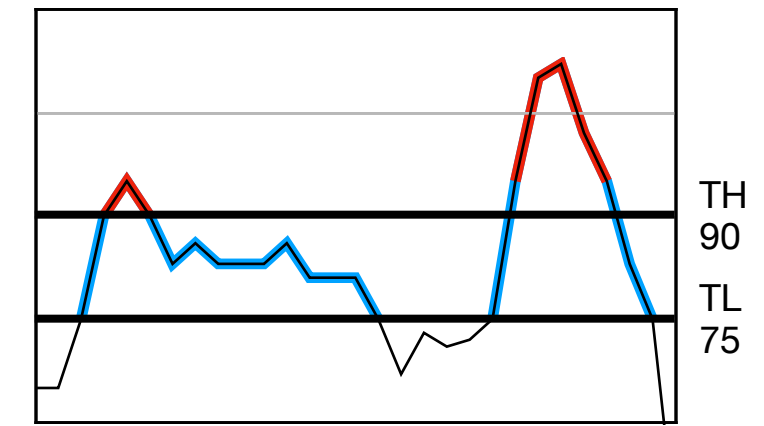
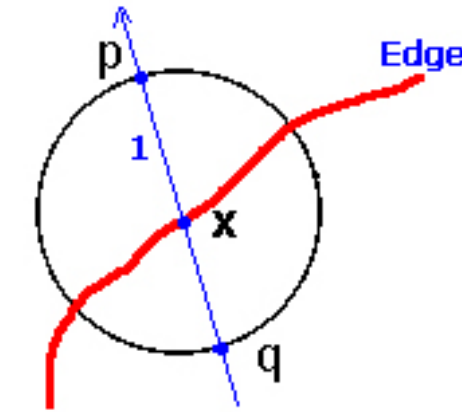
$\nabla f = (f_x, f_y)$

$\theta = \arctan(f_y/f_x)$

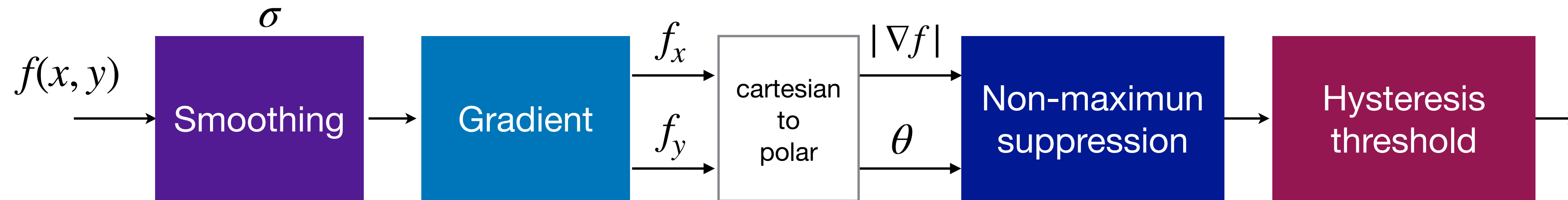
$\theta = \arctan(f_y/f_x)$

Edge Detection

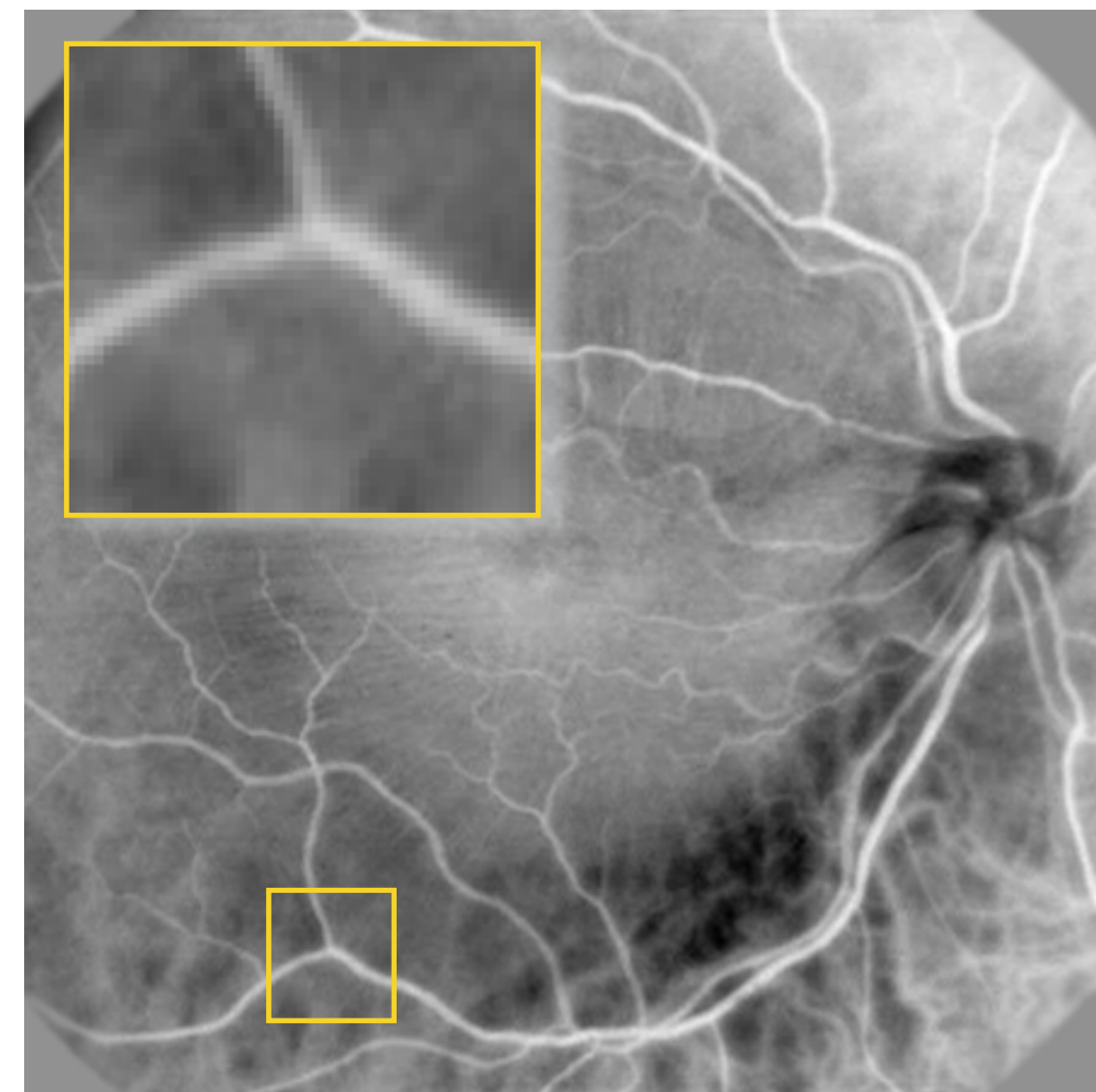
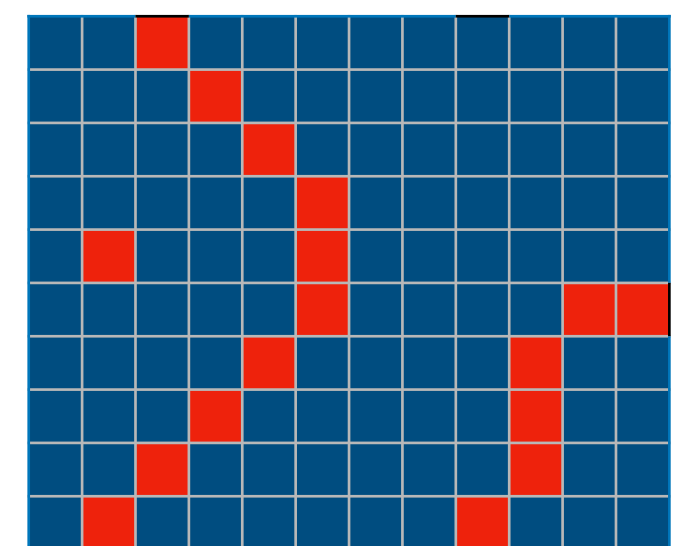
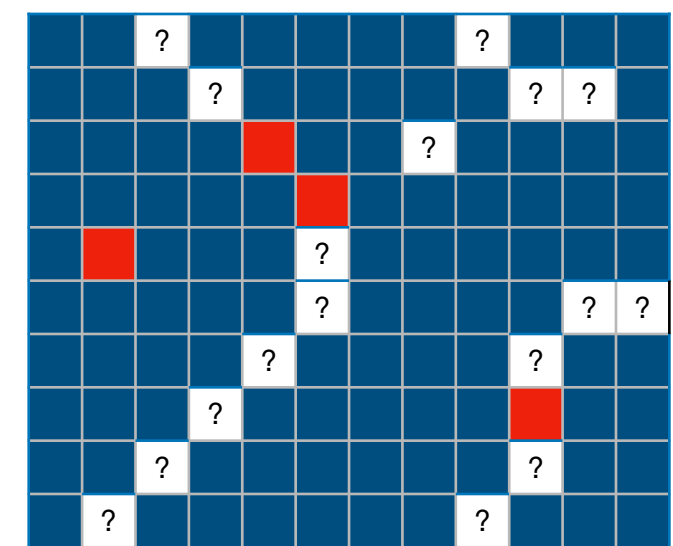
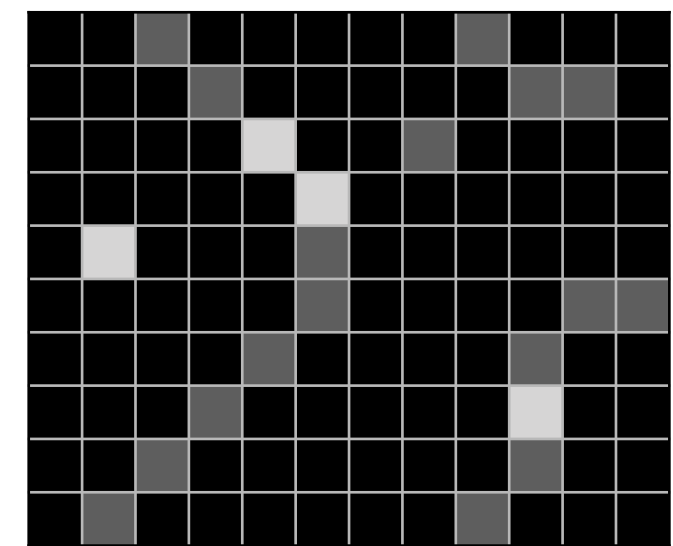
Canny edge detector [Canny 1986]



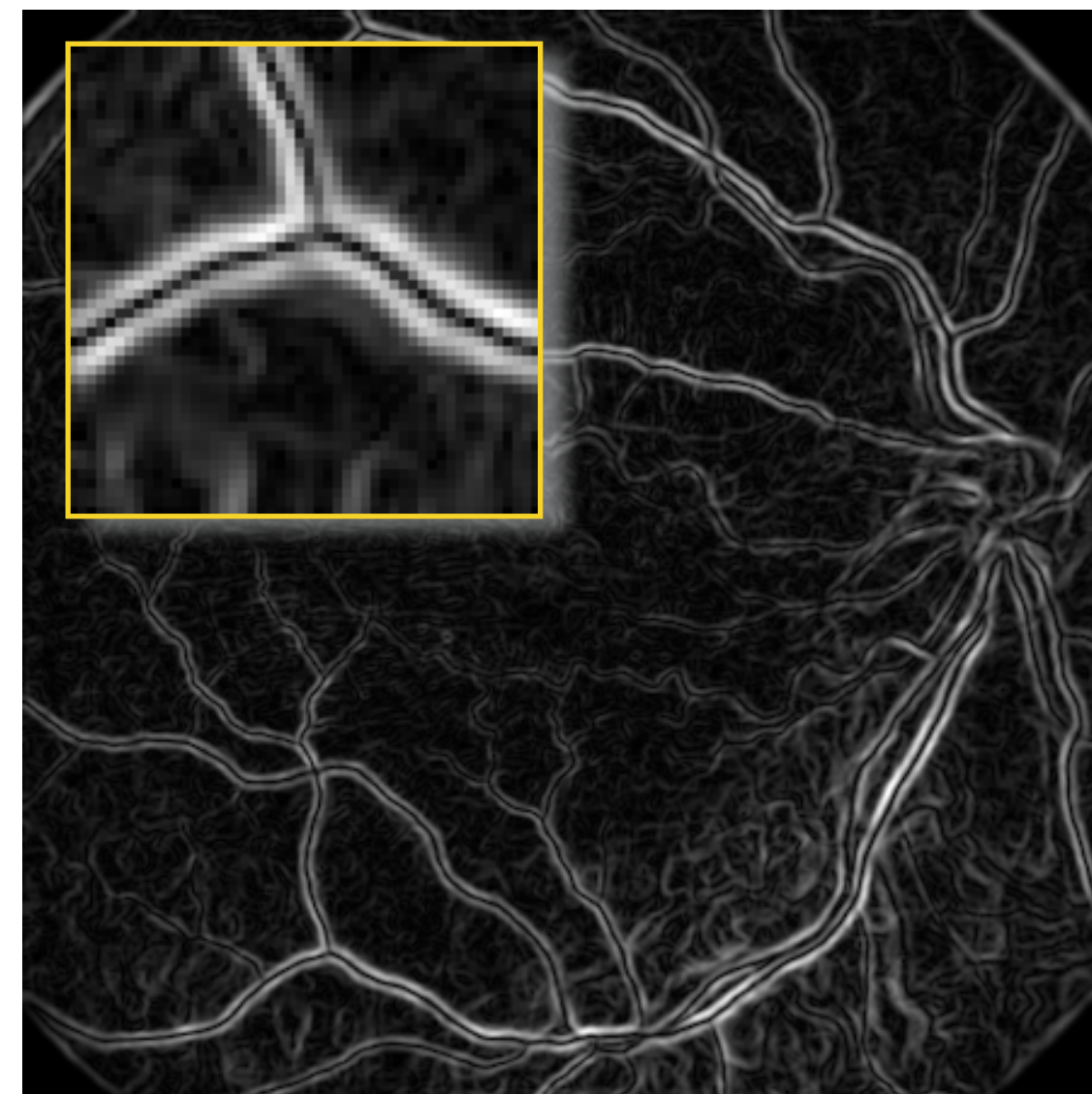
$T_{\text{low}}, T_{\text{high}}$



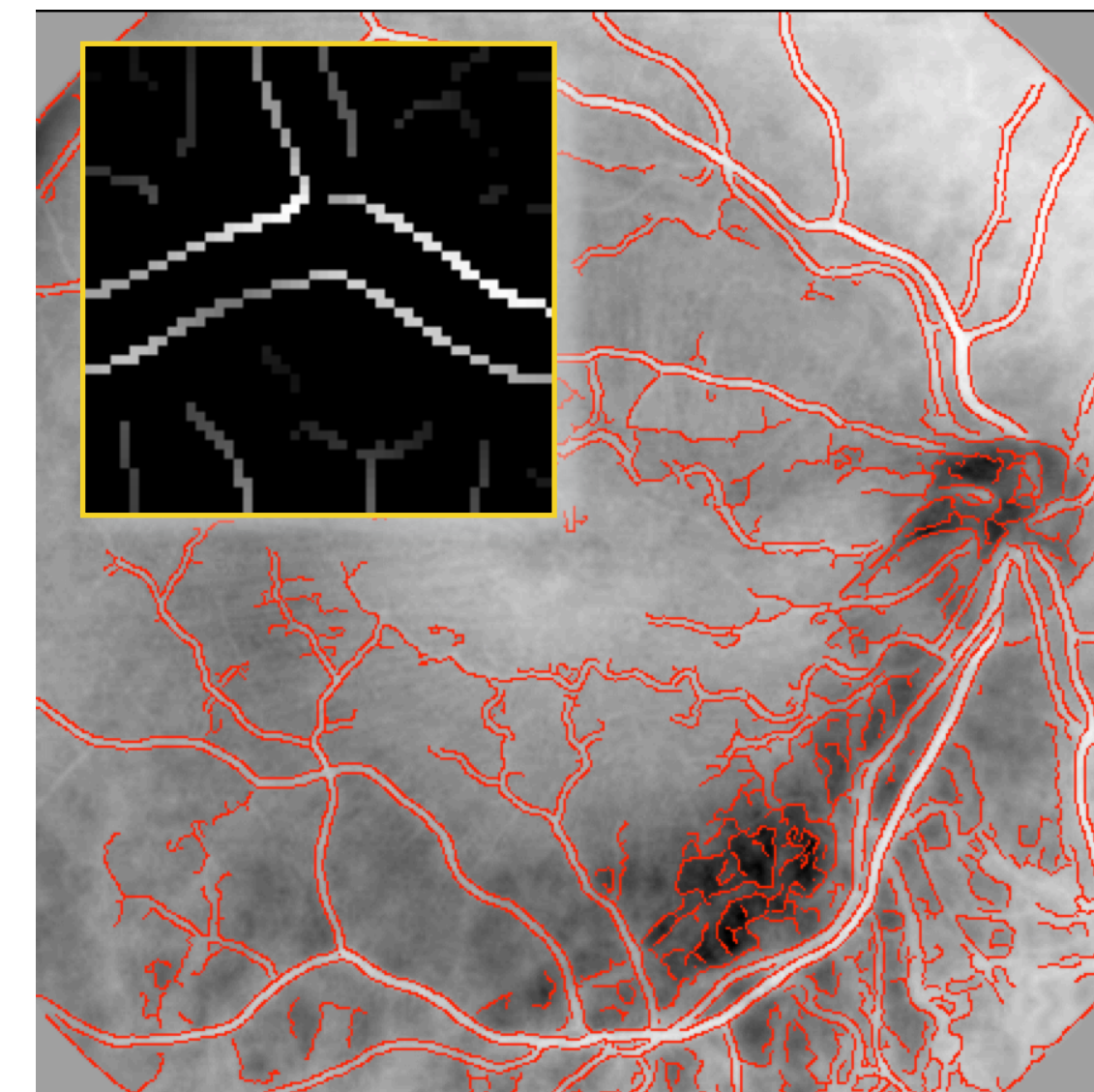
$$|\nabla f| = \sqrt{f_x^2 + f_y^2}$$



Input: fundus

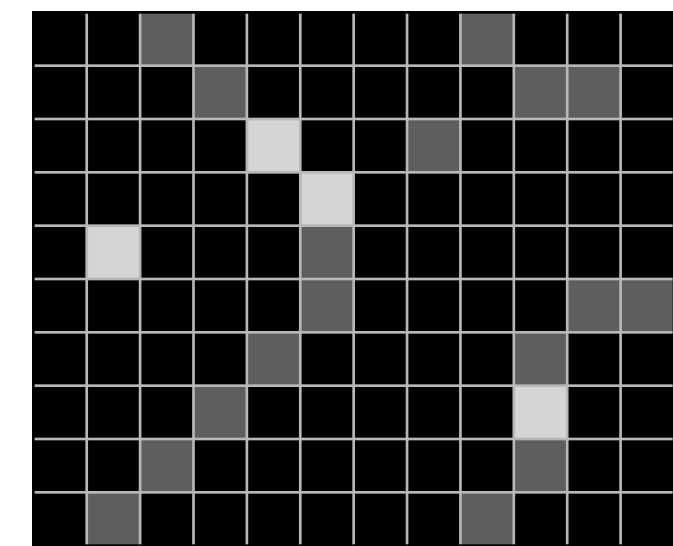
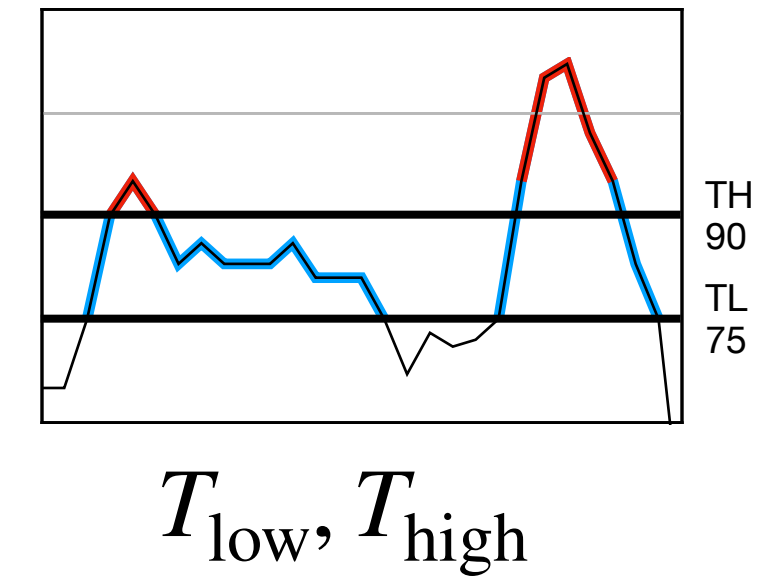
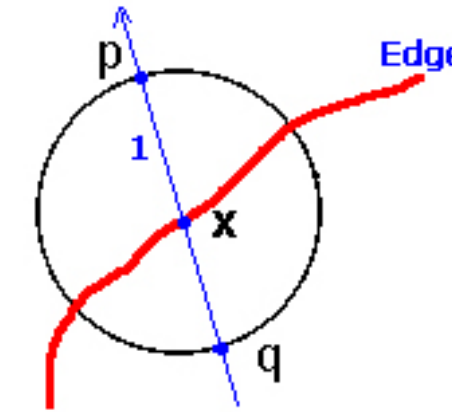
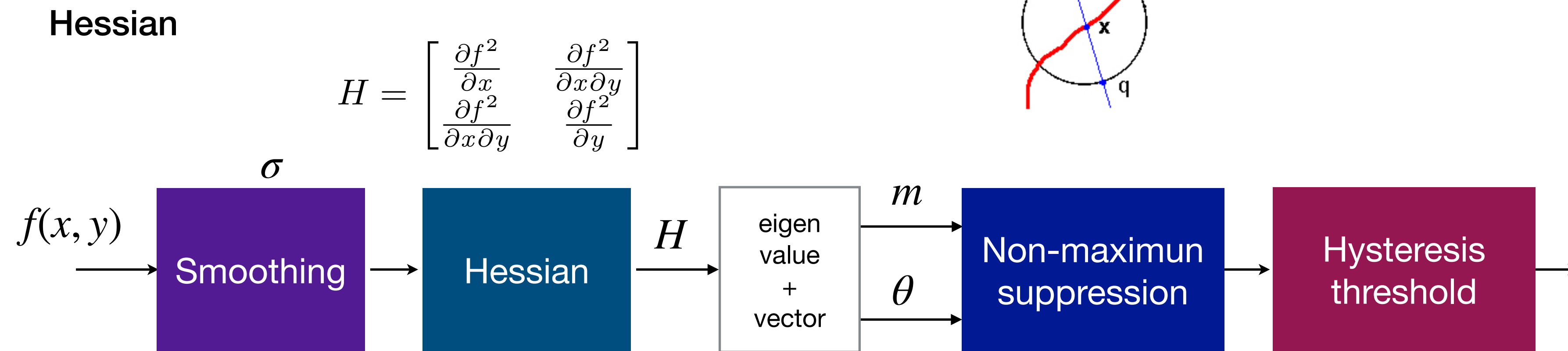


Gradient



Detected edges

Ridge Detection



$$m = \sqrt{\lambda_1} \sqrt{\lambda_1 - \lambda_2}$$



Input: fundus

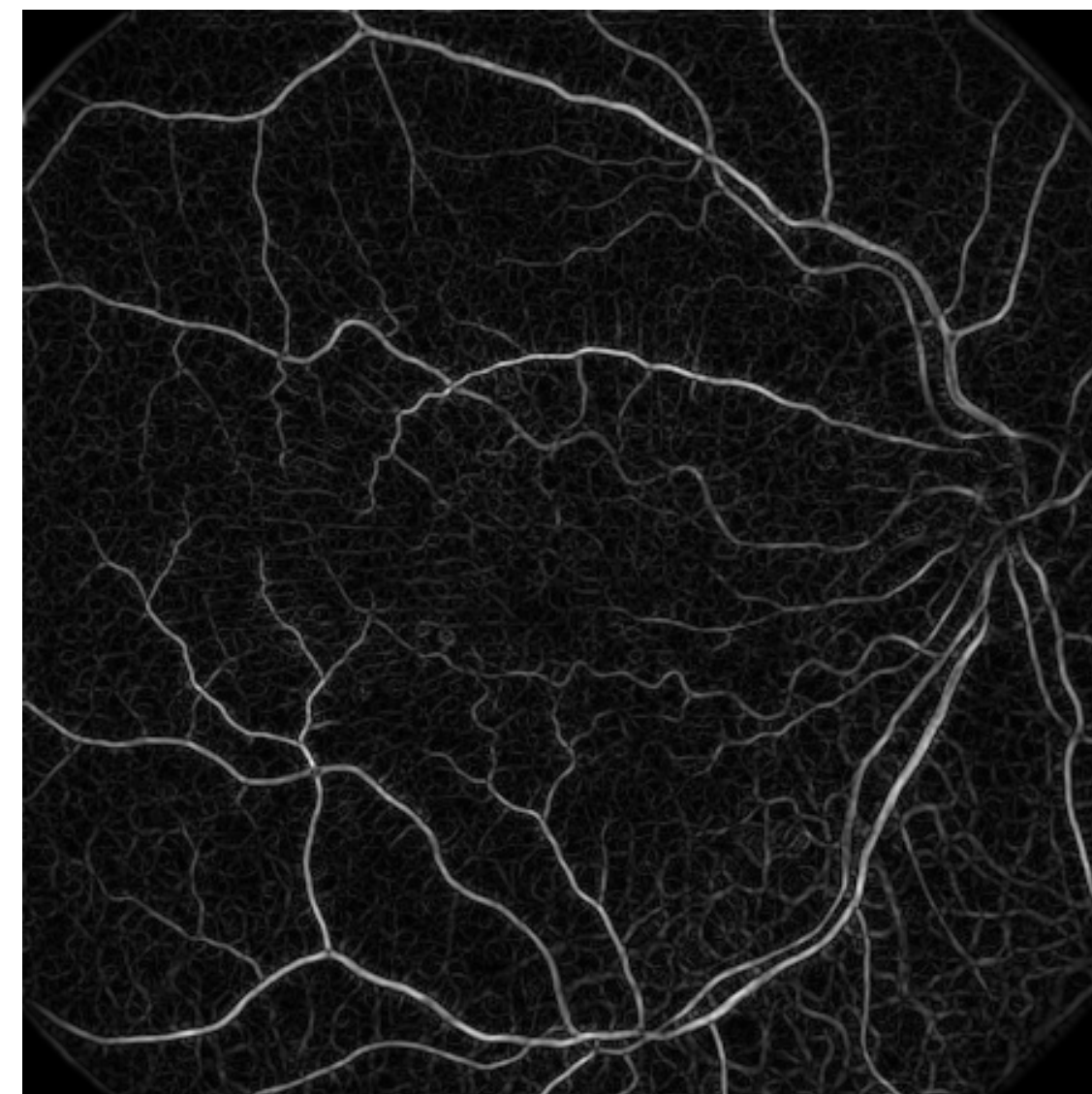
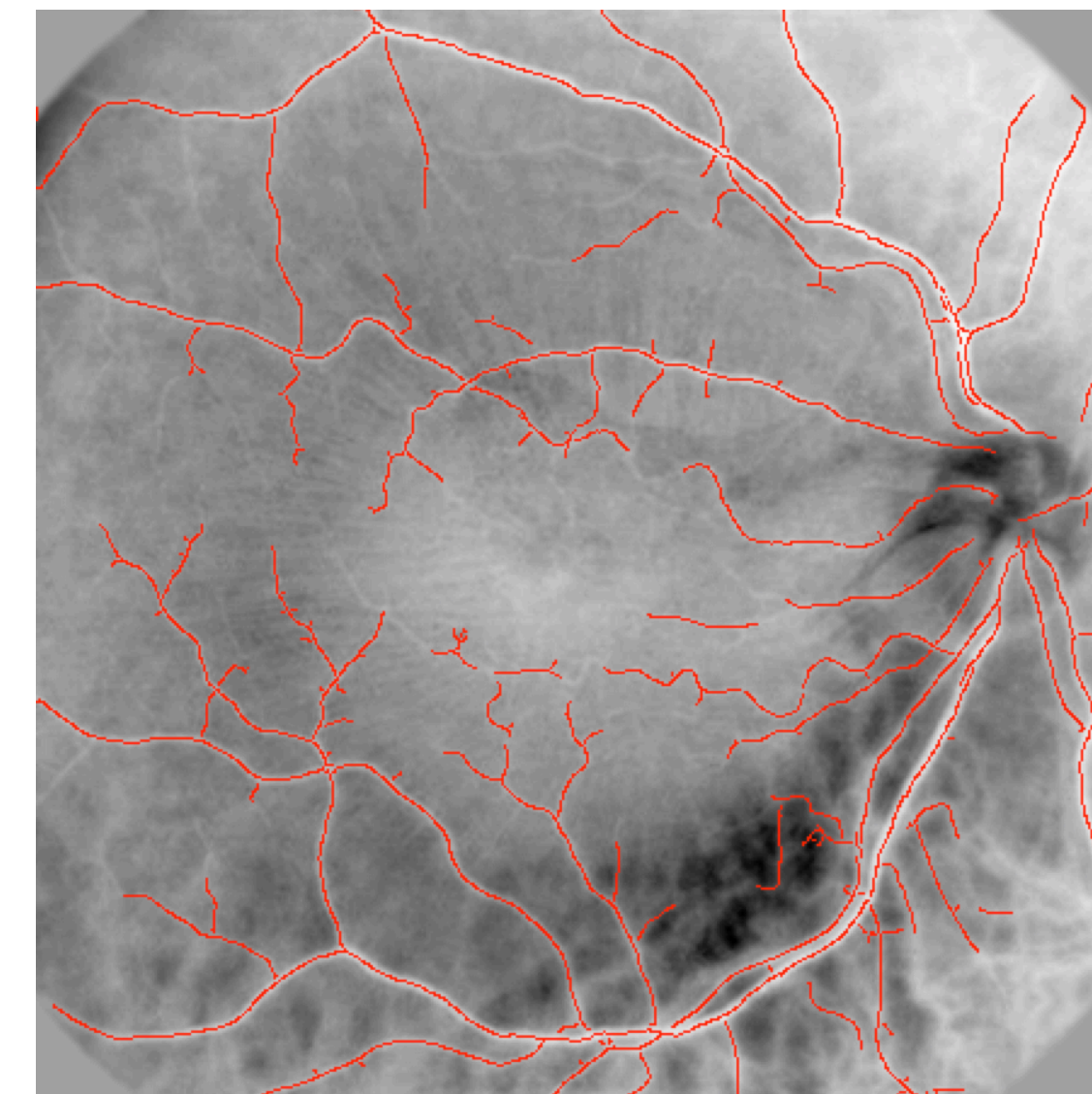
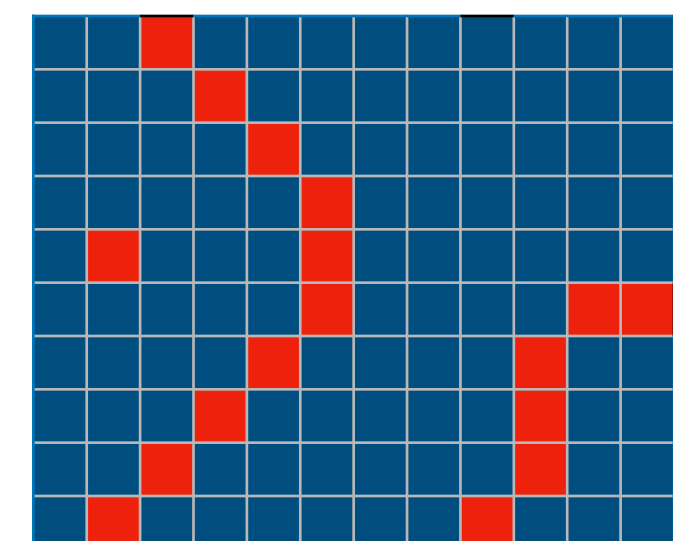
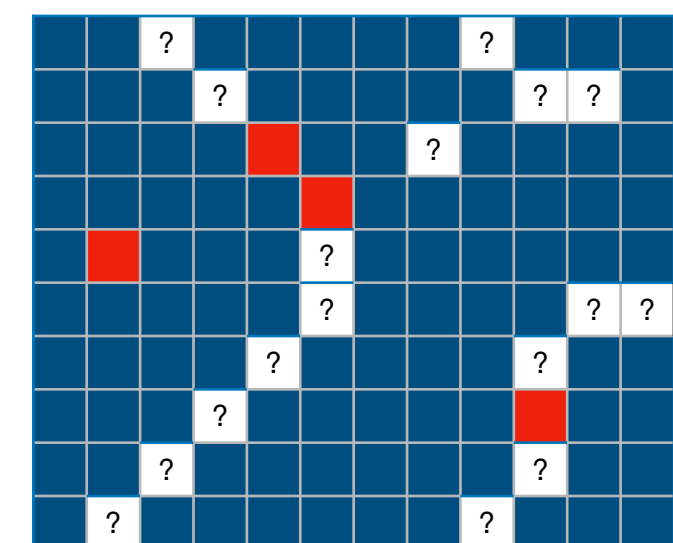


Figure of merit



Detected ridges



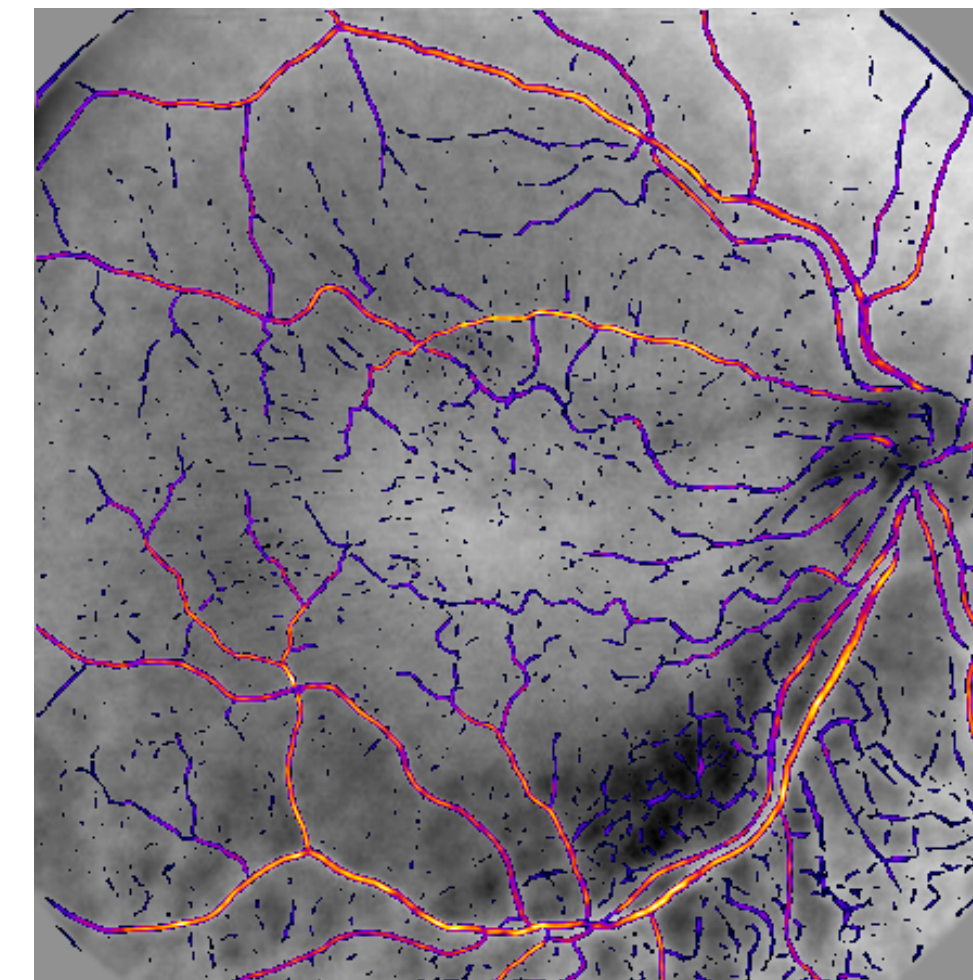
Filament-like, Membrane-like Detection



Input: fundus

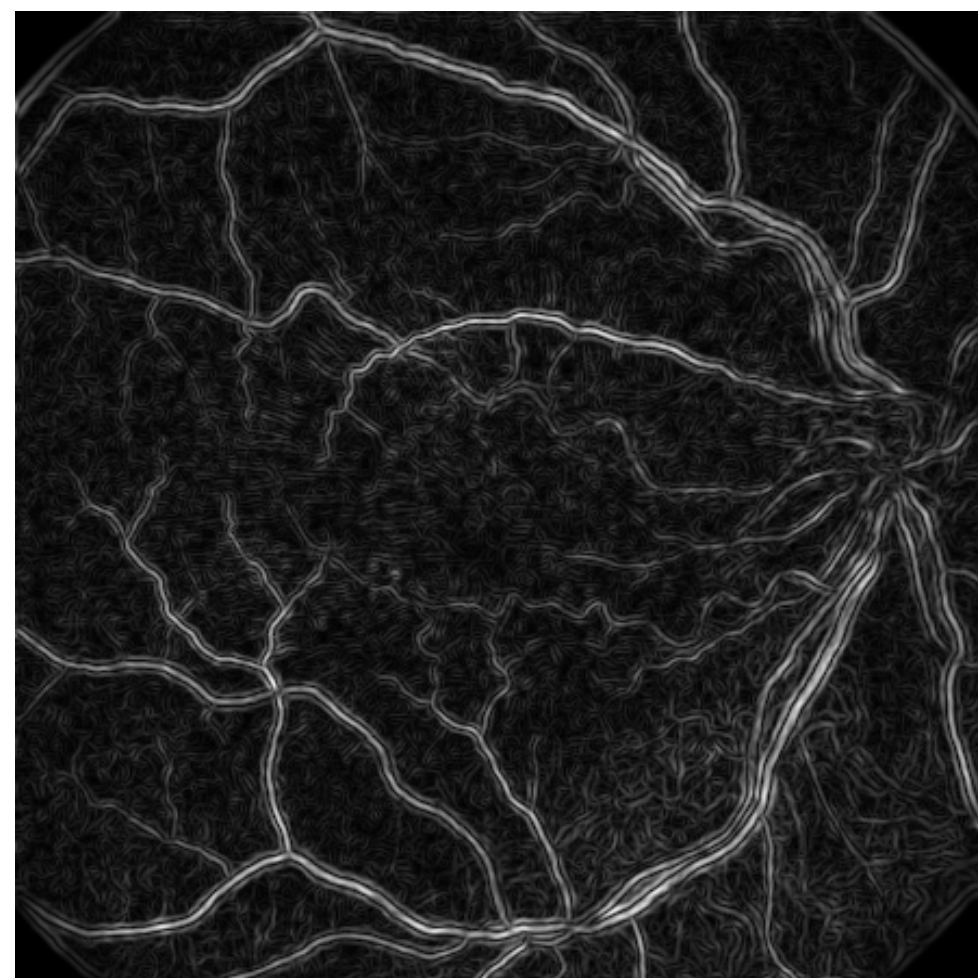


Threshold + Skeletonization



Vesselness (FIJI)

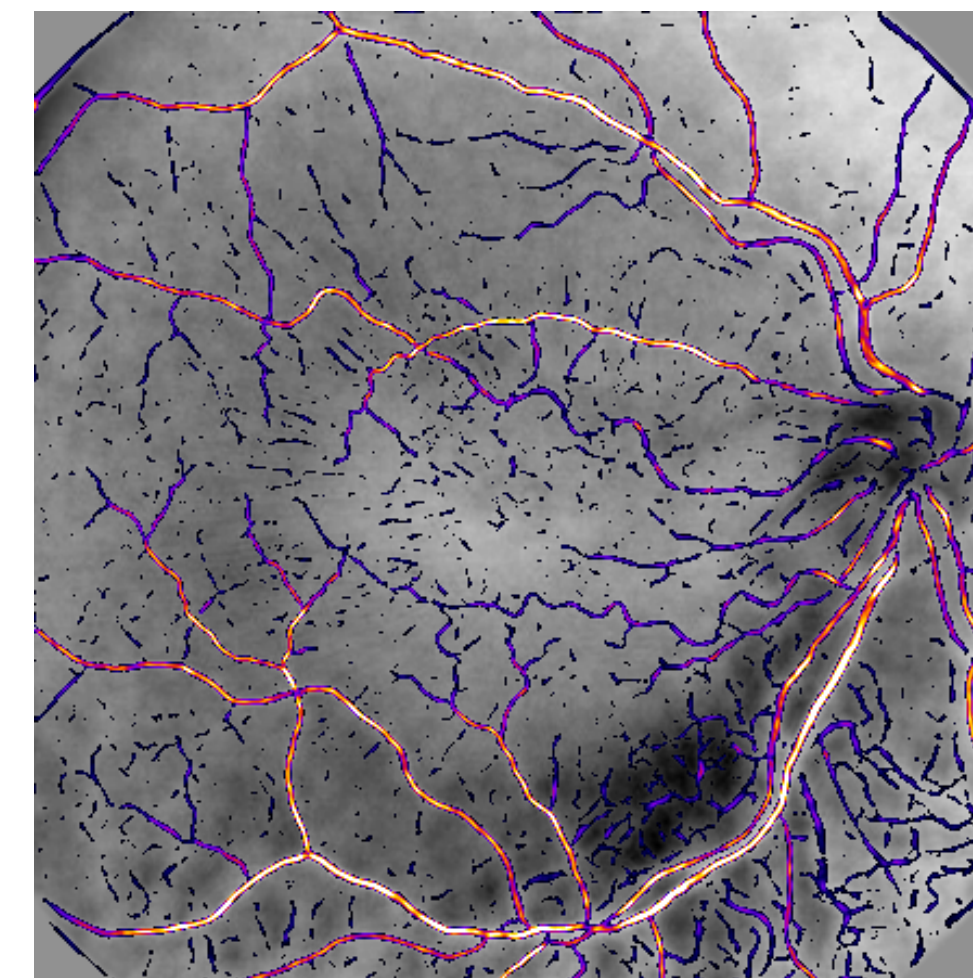
Video



Largest Hessian eigenvalues

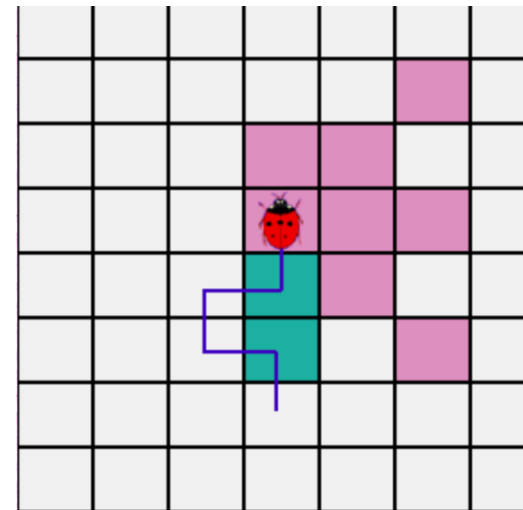


Canny detector for ridges

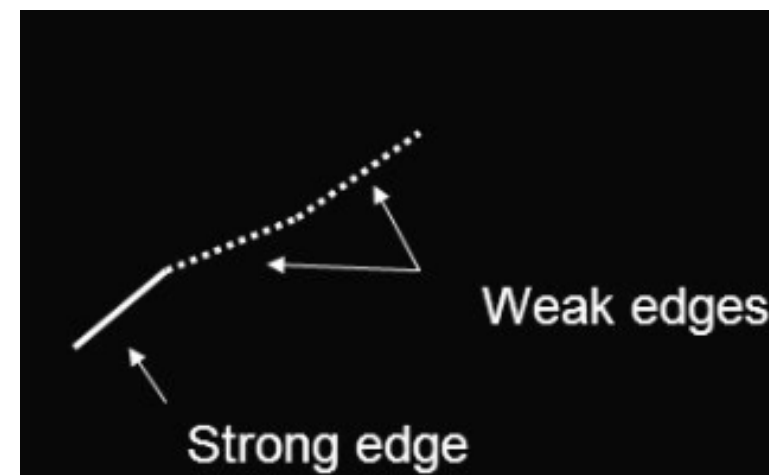


Tubeness (FIJI)

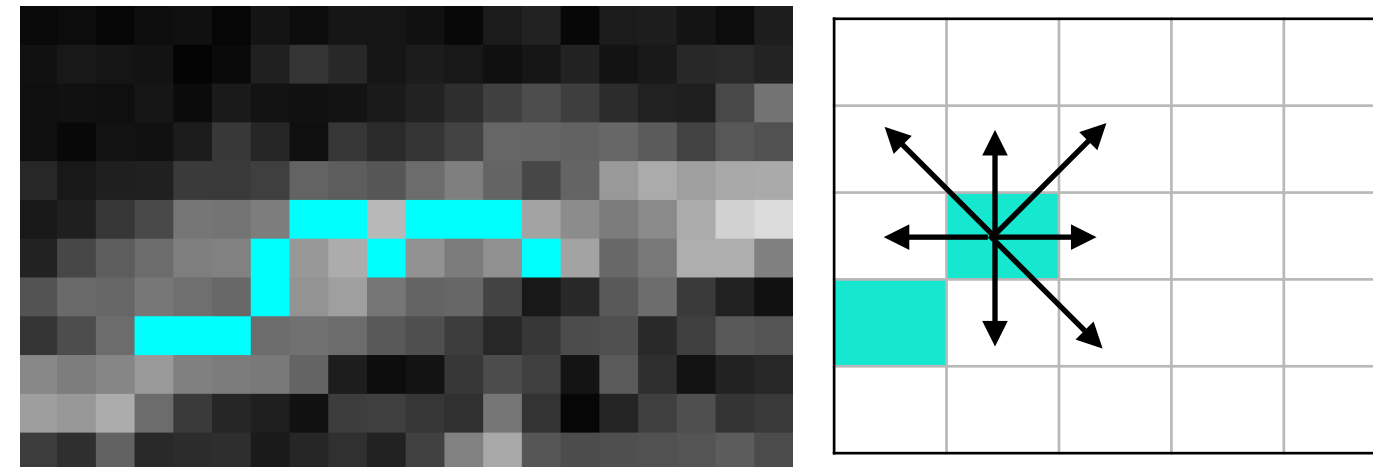
Square tracing algorithm “left-right”



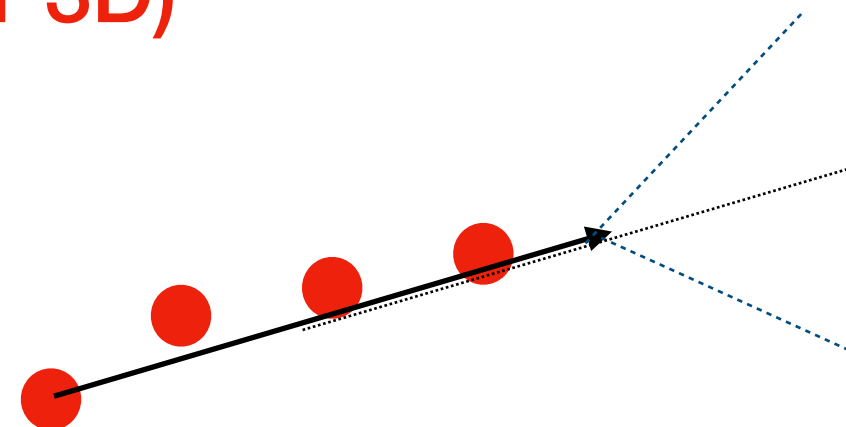
Low and high thresholds



Follow the local extrema



Using magnitude and direction



Viterbi Algorithm

Backtrack the shortest path on a graph

-	-	A	T	C	G	A	C
-	0	-4	-8	-12	-16	-20	-24
C	-4	-3	-7	-3	-7	-11	-15
A	-8	1	-3	-7	-6	-2	-6
T	-12	-3	6	2	-2	-6	-5
A	-16	-7	2	3	-1	3	-1
C	-20	-11	-2	-1	0	-1	8



Keypoint Detection

Corner Harris Detector

- Based on Gradient Structure Tensor
- Local maximum on Harris index

SIFT

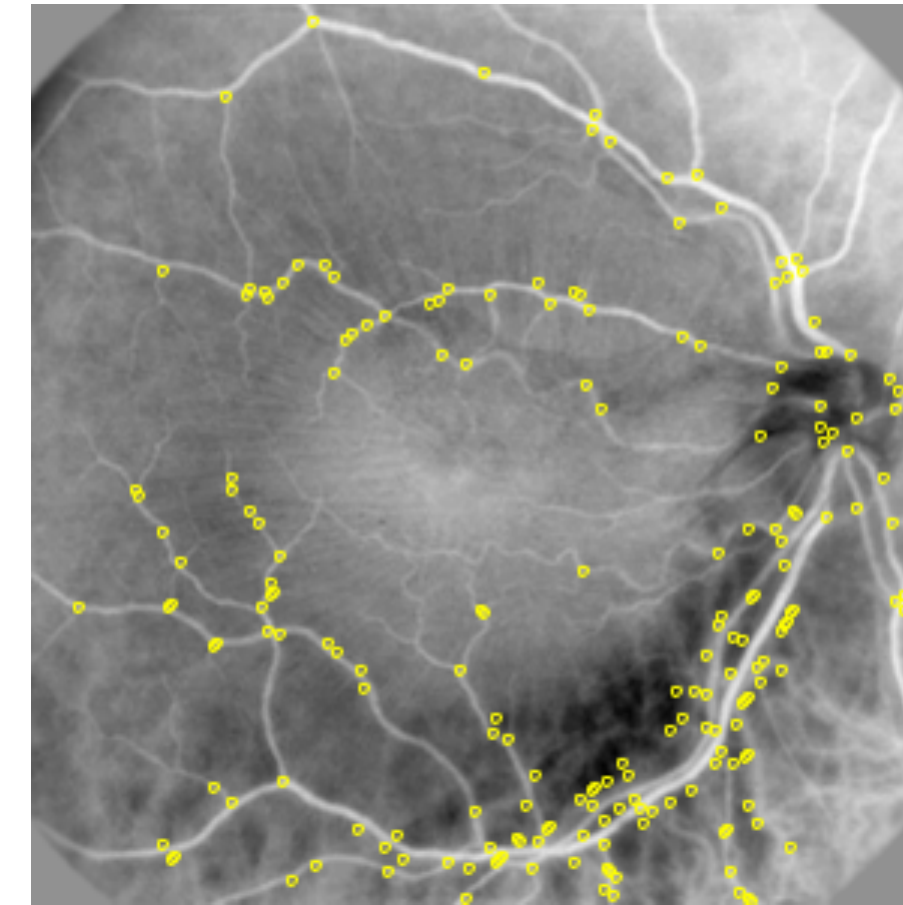
Scale-invariant Feature Transf. (H. Lowe, 2006)

- Scale invariant with series of DoG
- Dominant orientation of local keypoint

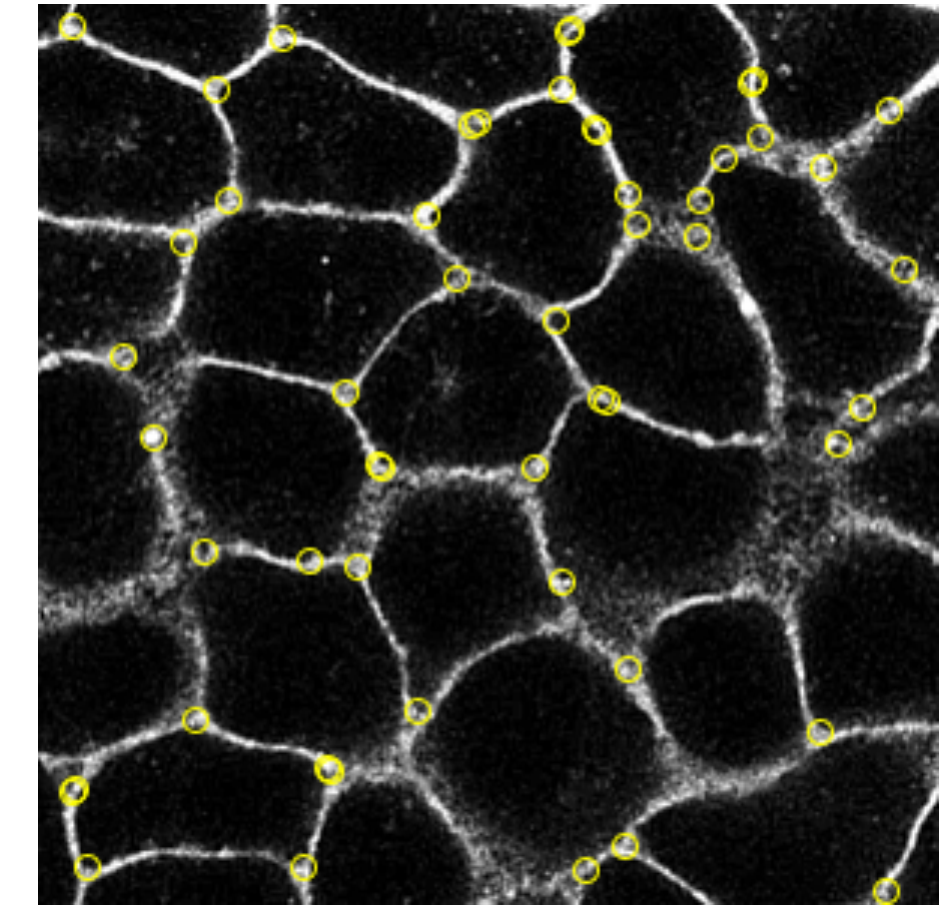
SURF

Speeded Up Robust Feature (H. Bays, 2006)

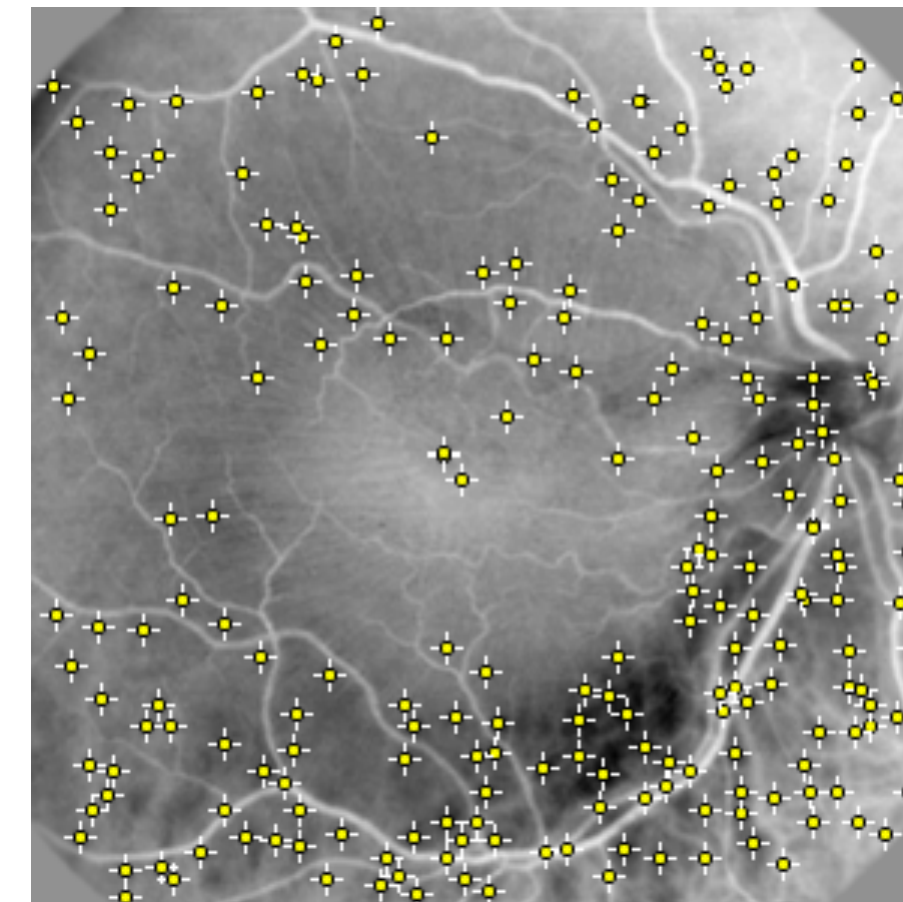
- Laplacian pyramid and gaussian pyramid
- Determinant of Hessian



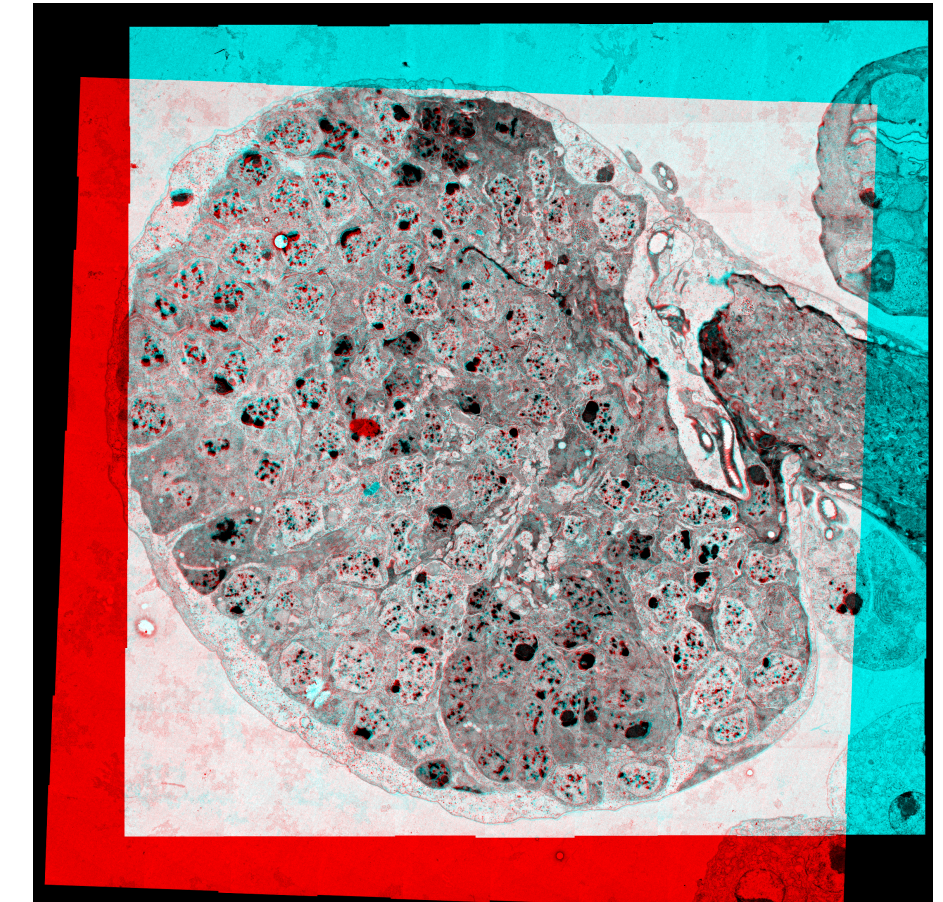
Harris Corner



Harris Corner



SIFT



Alignment, registration of 2 sets of keypoints

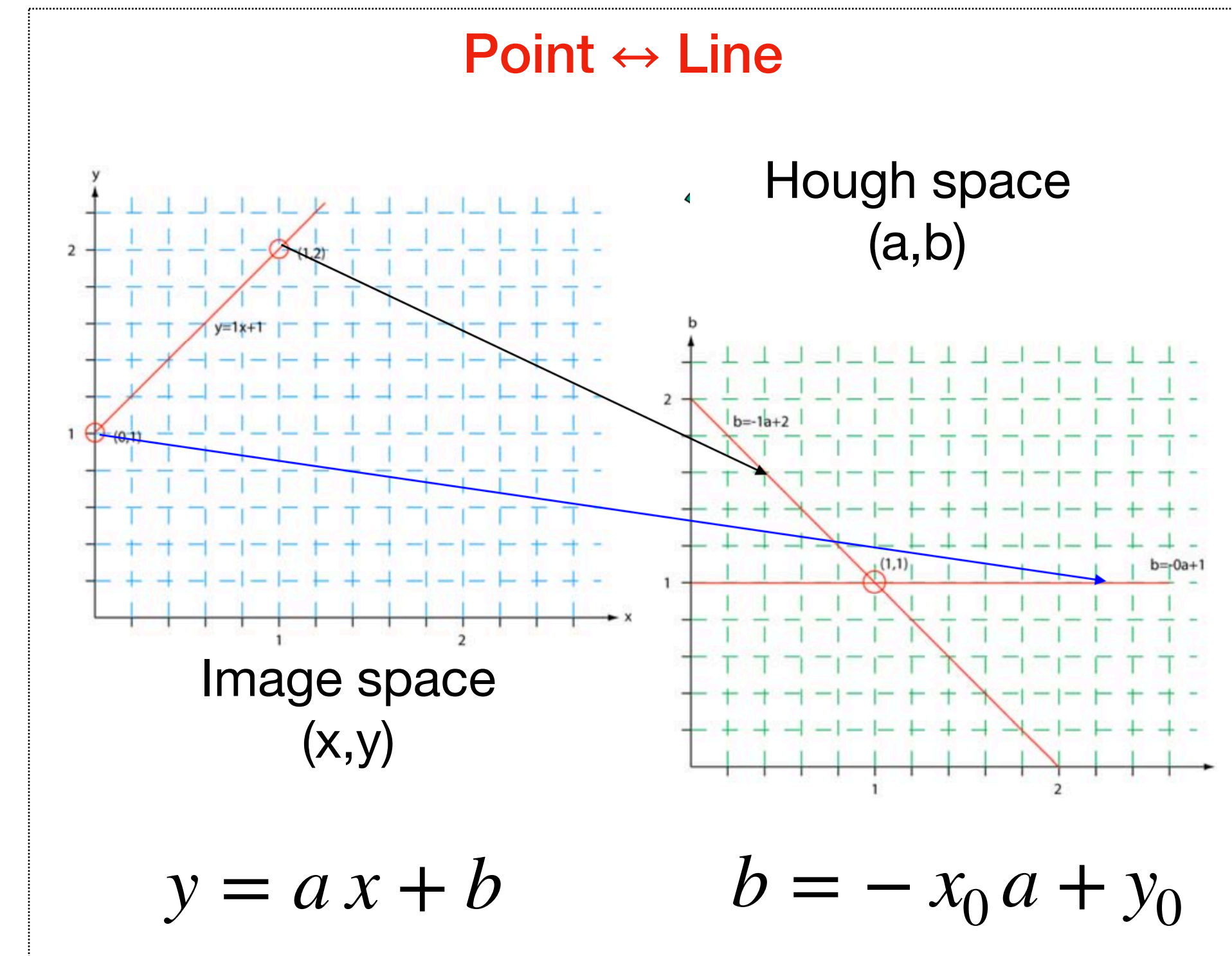
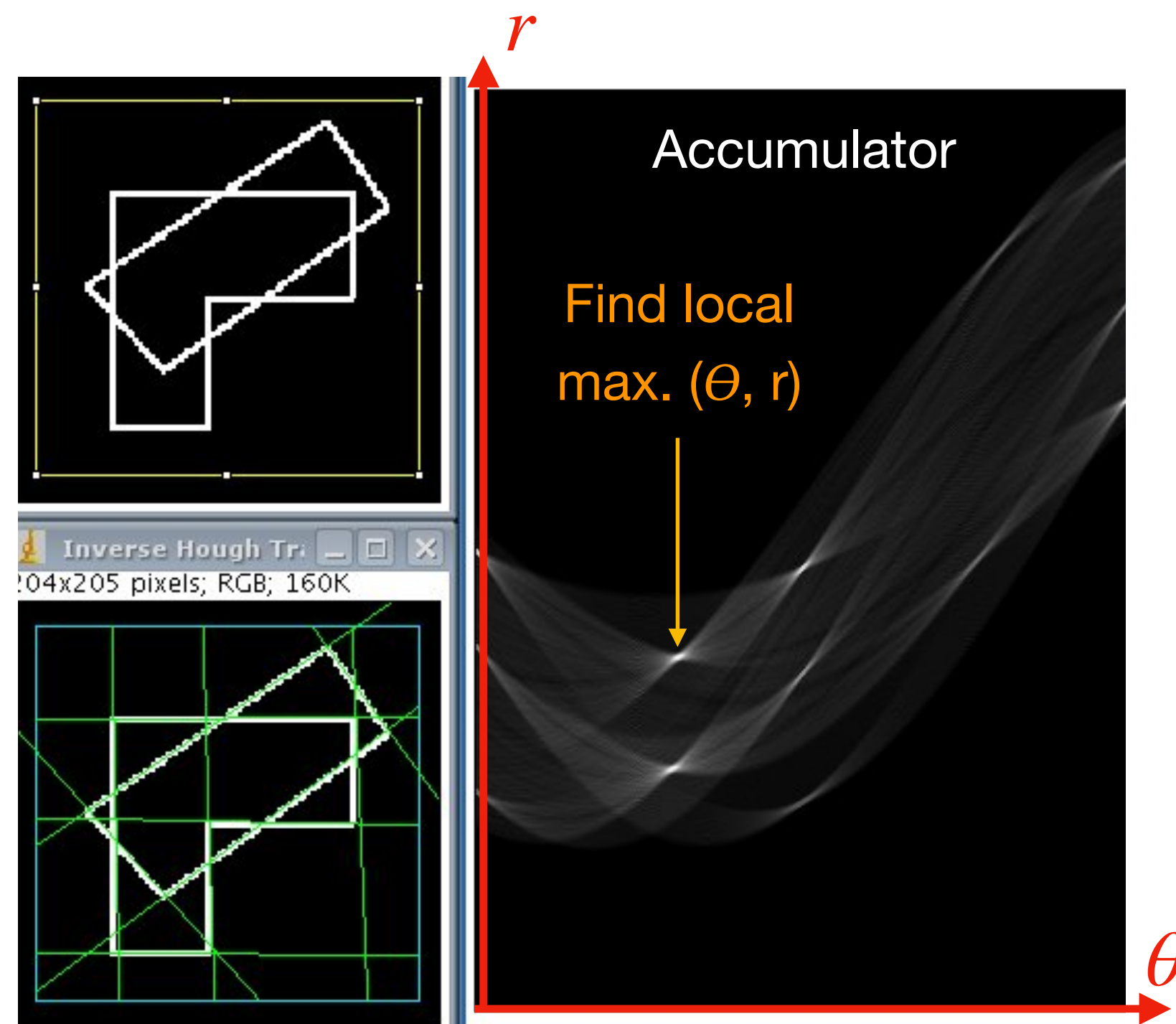
JavaSIFT, Stephan Saafeld

Line Detection Hough Transform

Principle

- Express the shape in the parameter space
- Voting procedure (slow)

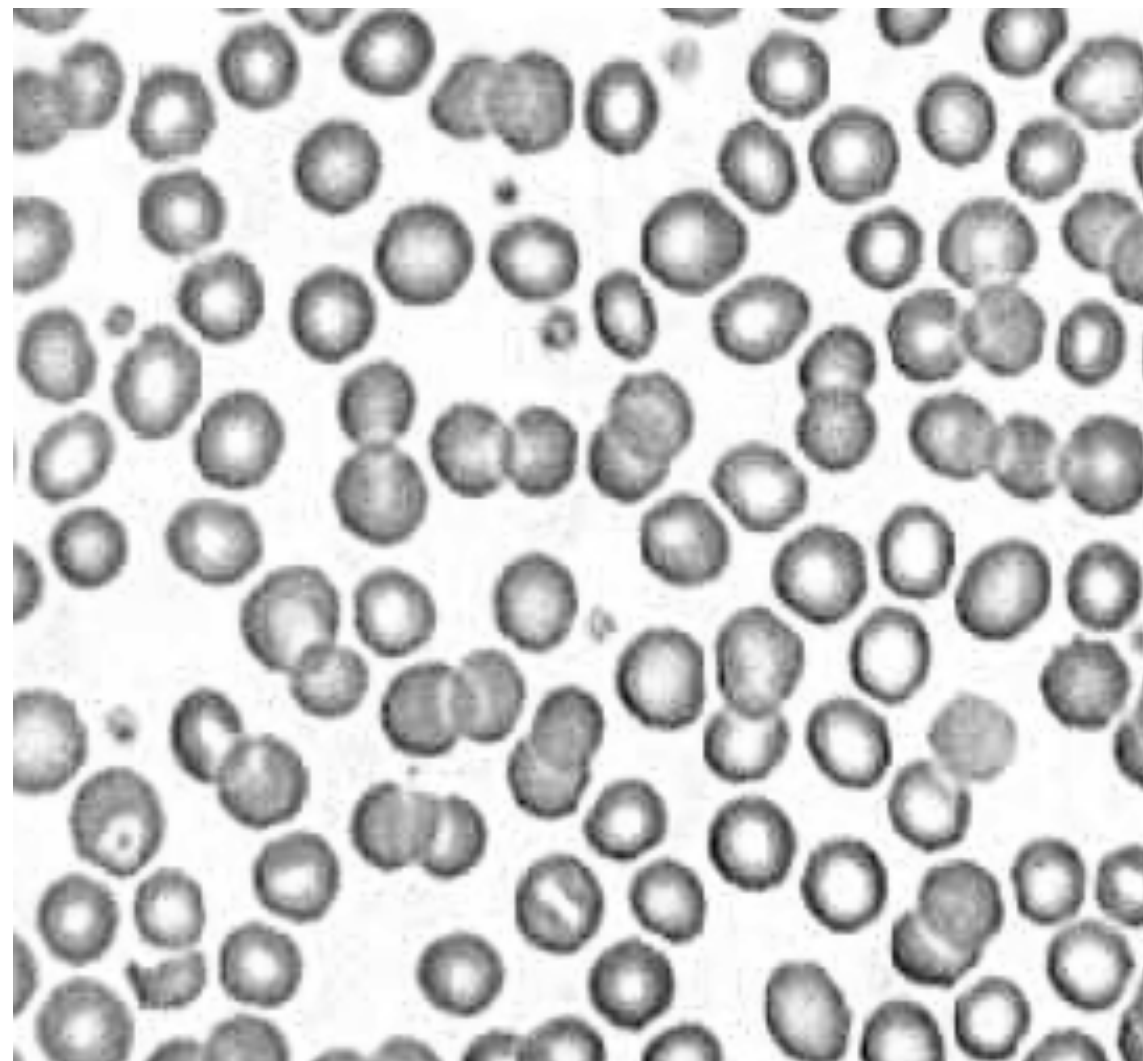
$$r = x_0 \cos(\theta) + y_0 \sin(\theta)$$



- ✓ Robust to noise
- ✓ Able to find approximative line
- ▶ Slow
- ▶ Discretization
- ▶ Threshold

👁 Circle Detection Generalized Hough Transform

Input
image



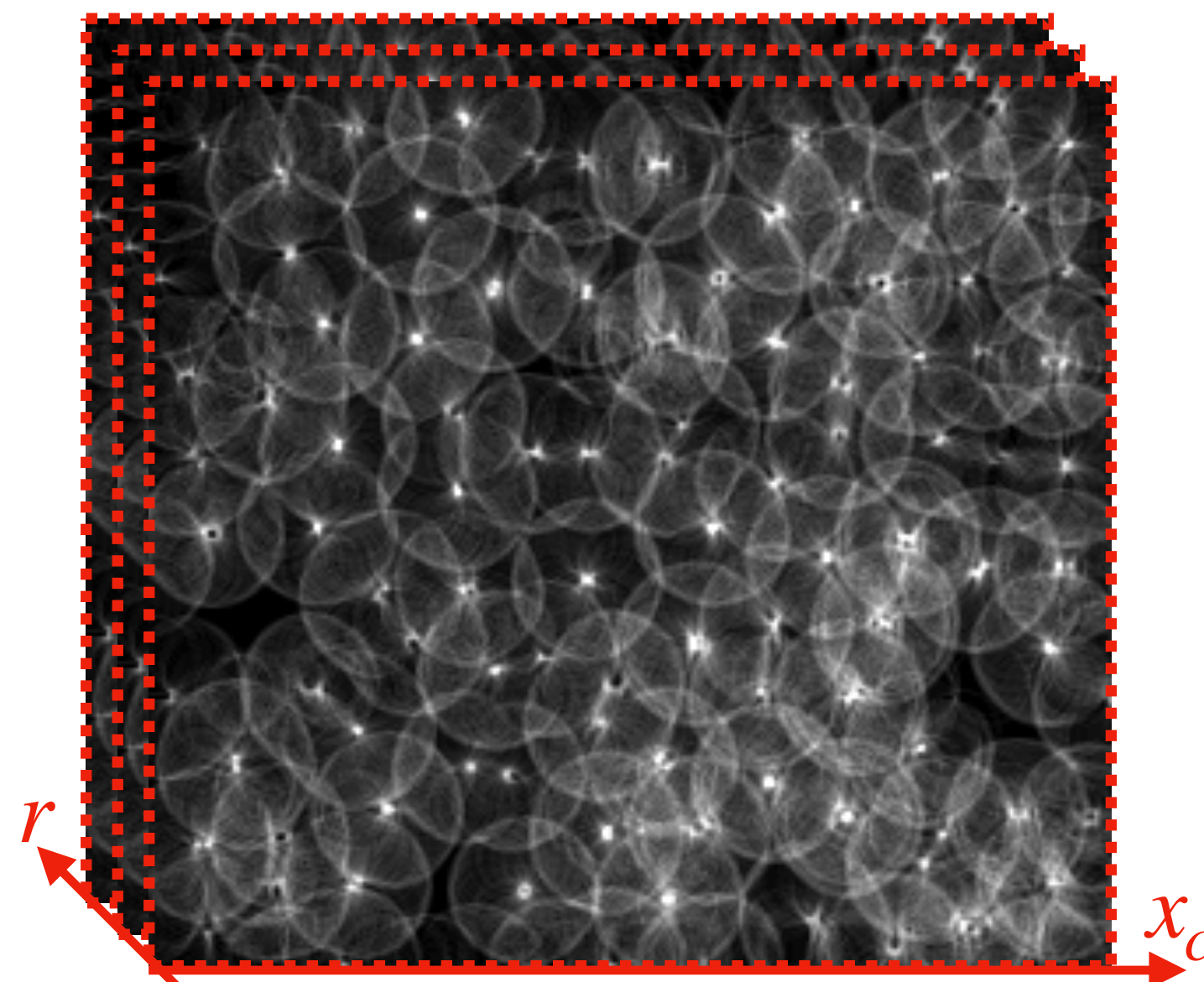
Circle

$$r^2 = (x - x_c)^2 + (y - y_c)^2$$

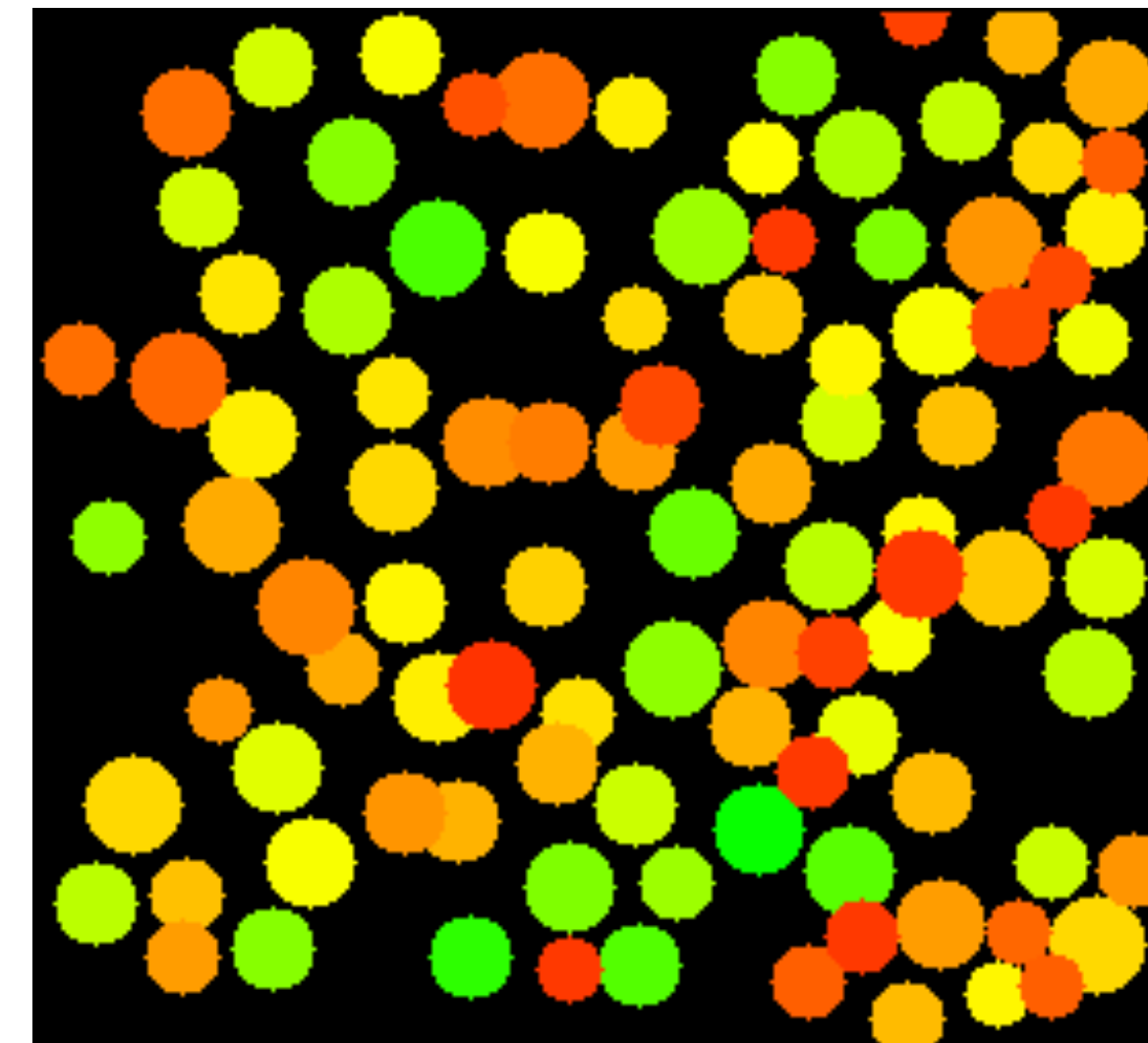
Hough space:

3 parameters (x_c , y_c , r)

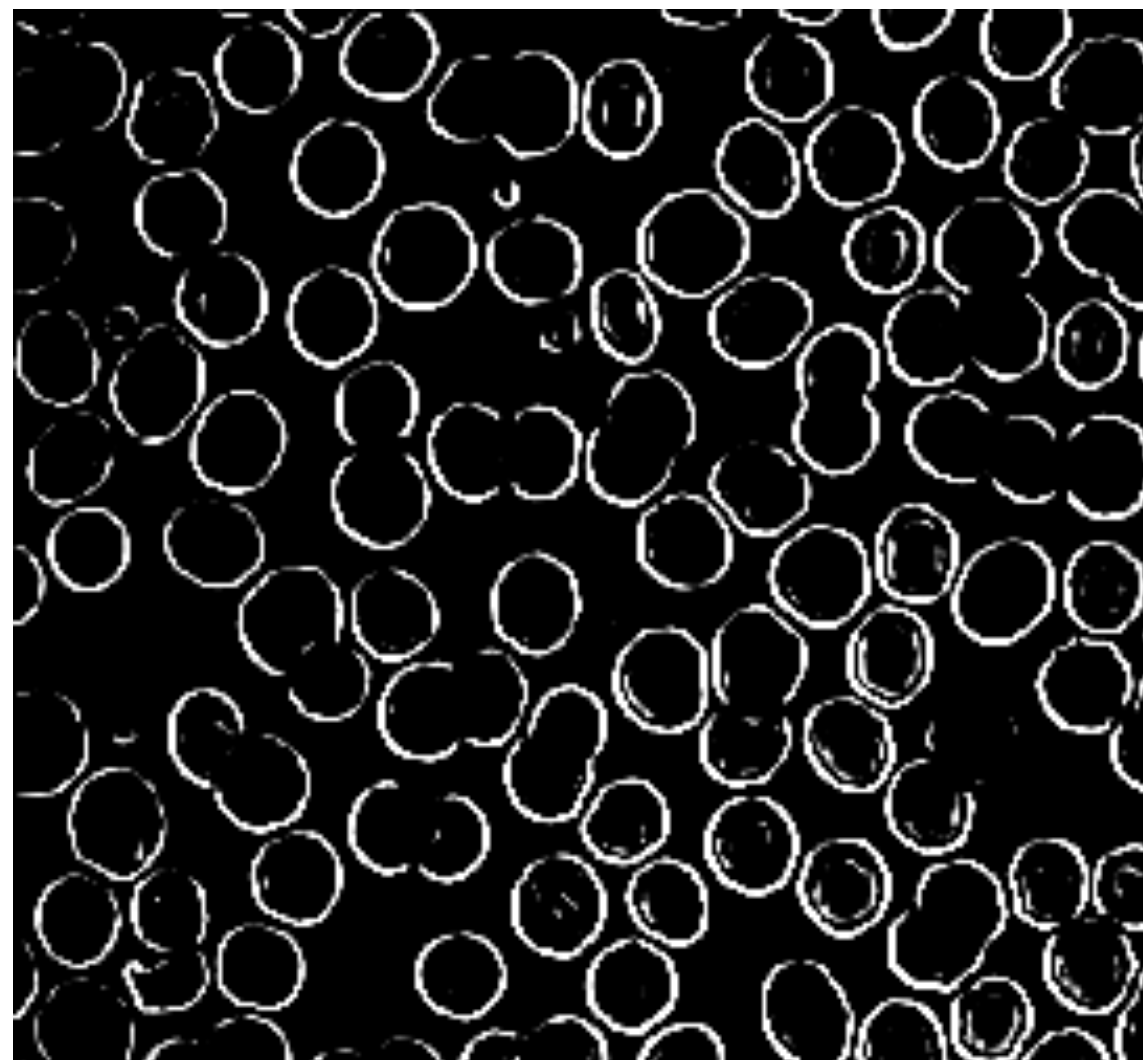
Accumulator



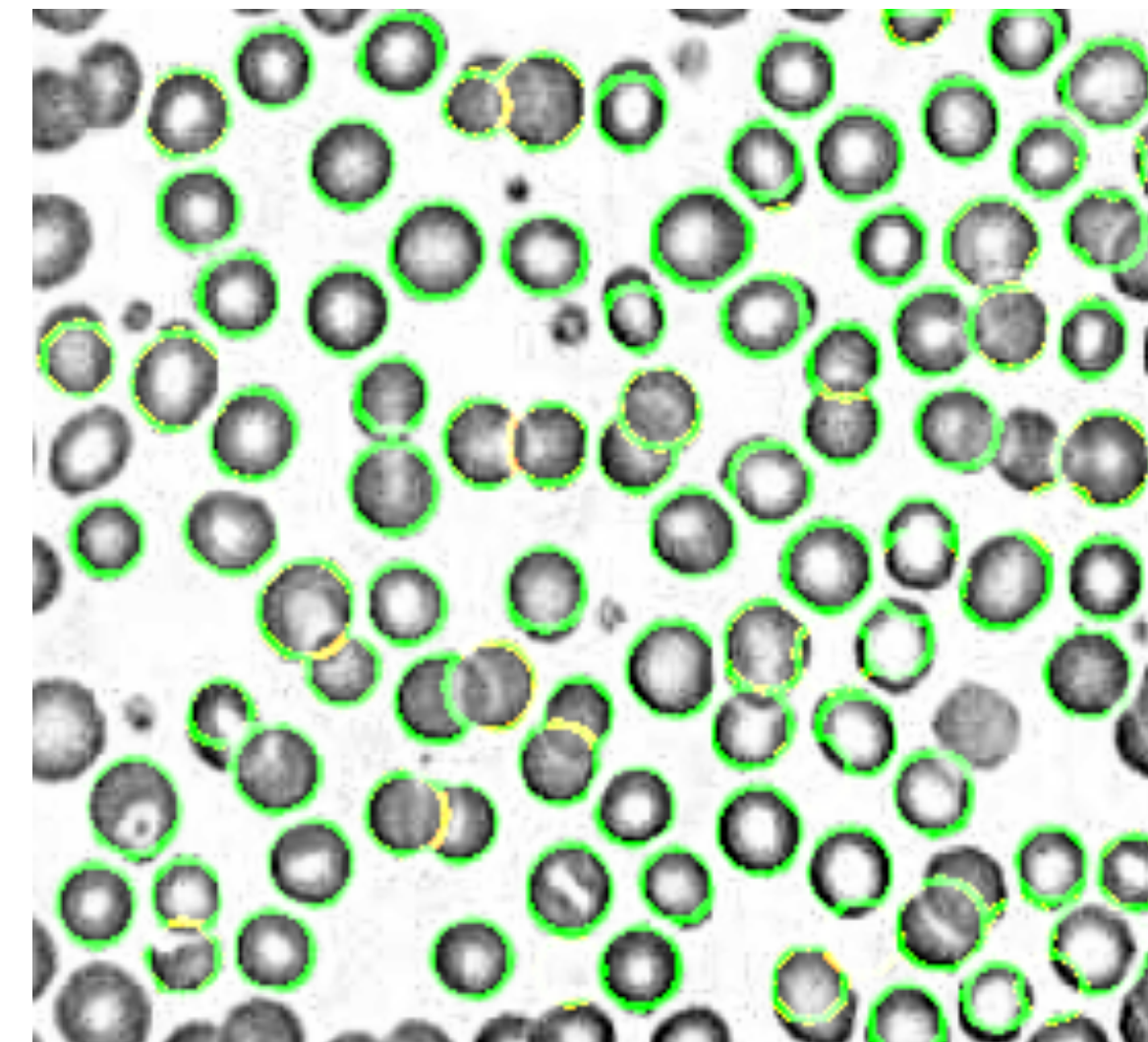
Score in
Hough Space



Edges
(Sobel)



Detection
(100 max.)



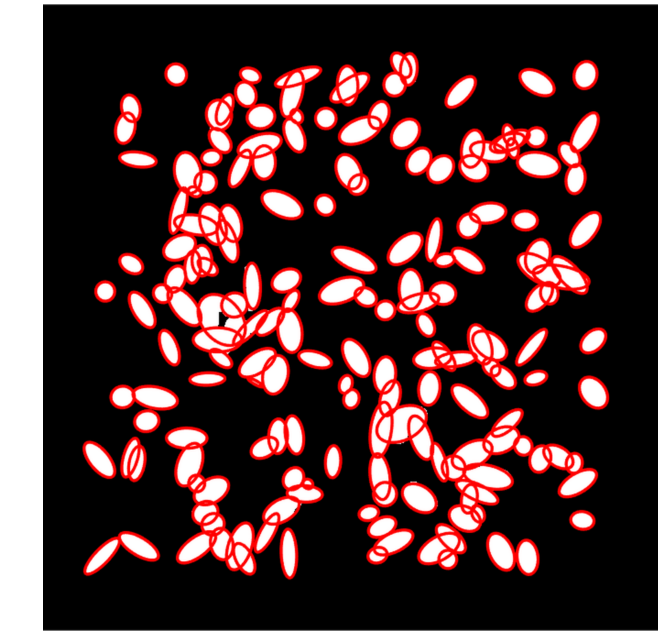
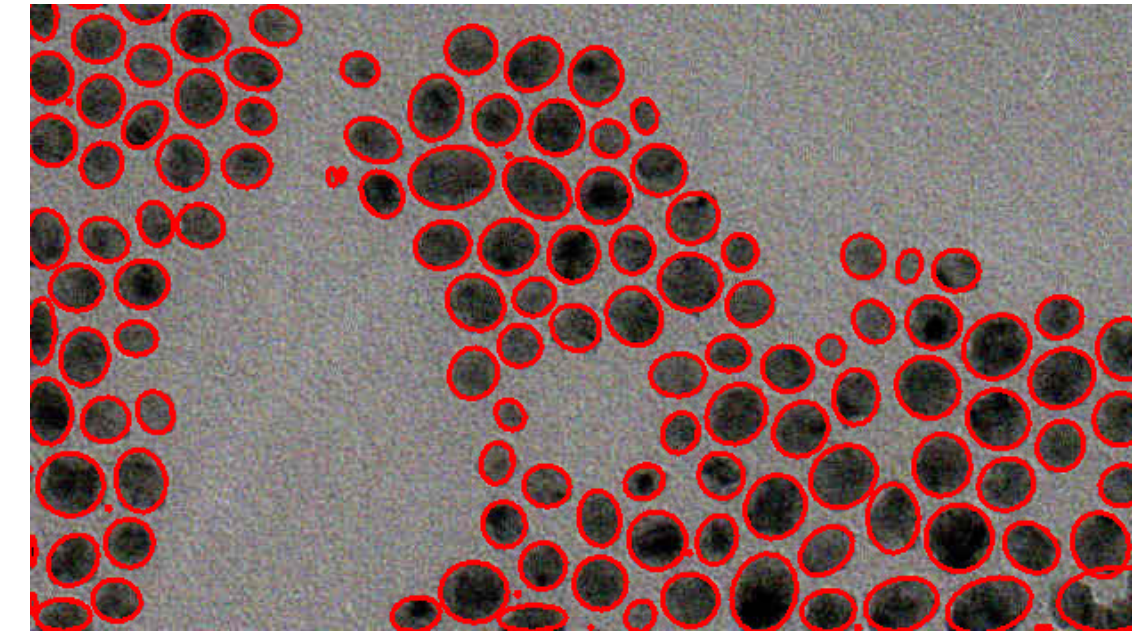


Parametric Shape Fitting

Ellipse curve fitting

on the contour image

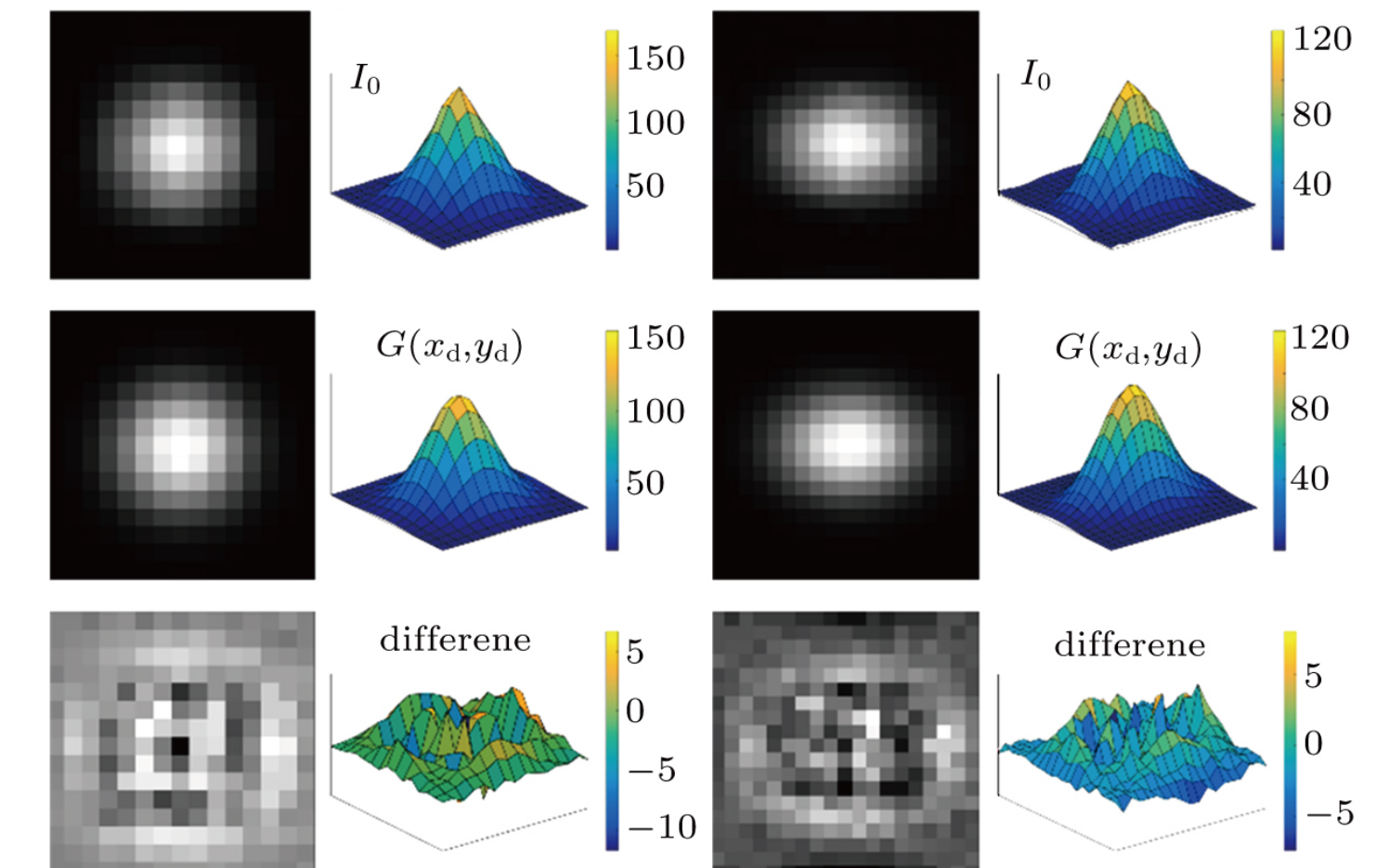
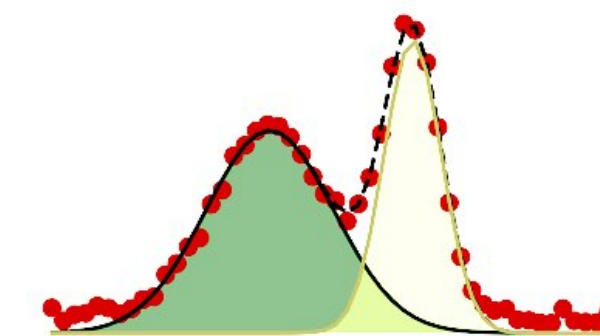
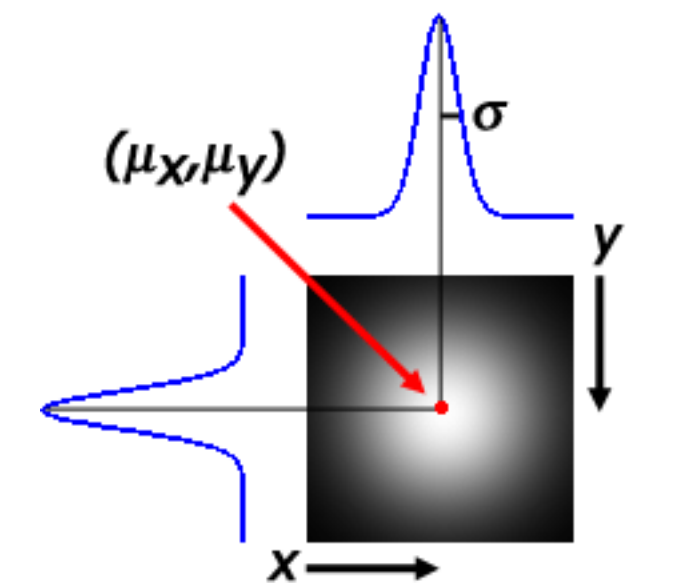
- Parameters: Location (x,y) - Axis - Orientation
- Required an initialization
- Overlap



Gaussian fitting

on the data image

- Parameters: Amplitude - Location - Sigmas - Orientation
- Required an initialization
- Overlap



Parametric curve fitting

→ Active contour

- N points on the curve





Texture Analysis

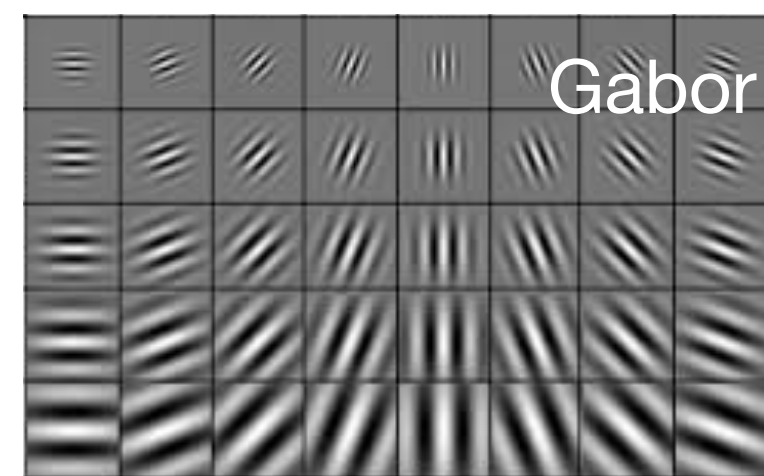
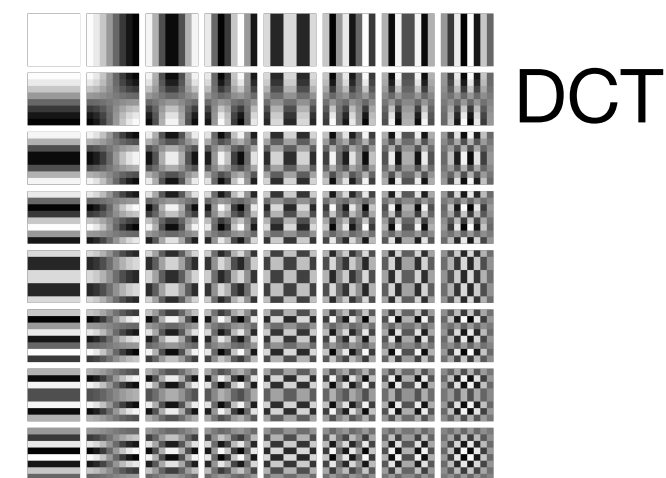
What is texture in image?

- **Structural** repeating pattern of local variations
- **Statistical** measure the arrangement of intensities in a region
- **Self-similarity** primitives with specific spatial distribution
- **Invariance** ✓ translation ✓ scale ✗ rotation

Texture operators

- Histogram
- Moments
- Co-occurrence matrix
- Gabor filters
- Filter bank (DCT)
- Wavelets
-

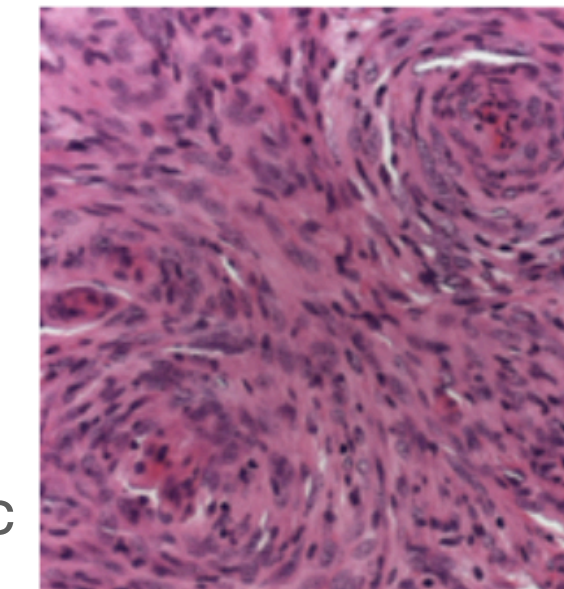
$$M_{p,q} = \iint x^p y^q f(x, y) \text{ Moments}$$



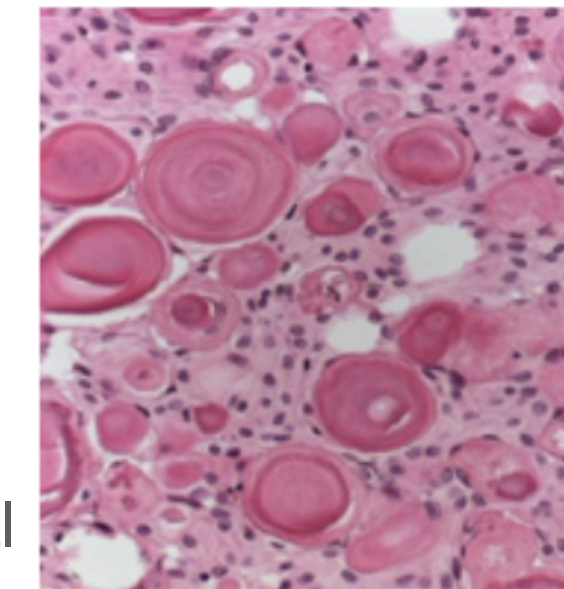
Color histopathology

3D X-ray CT scan

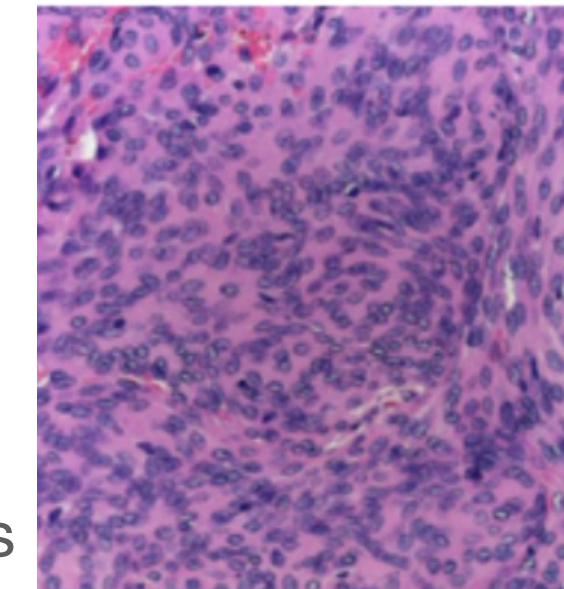
fibroblastic



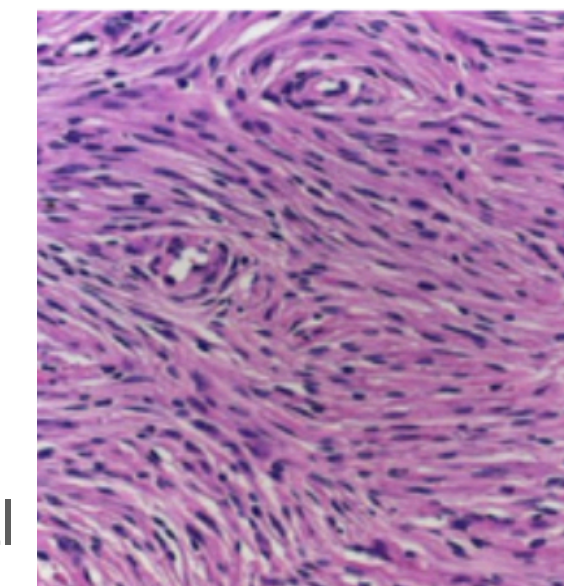
meningotheial



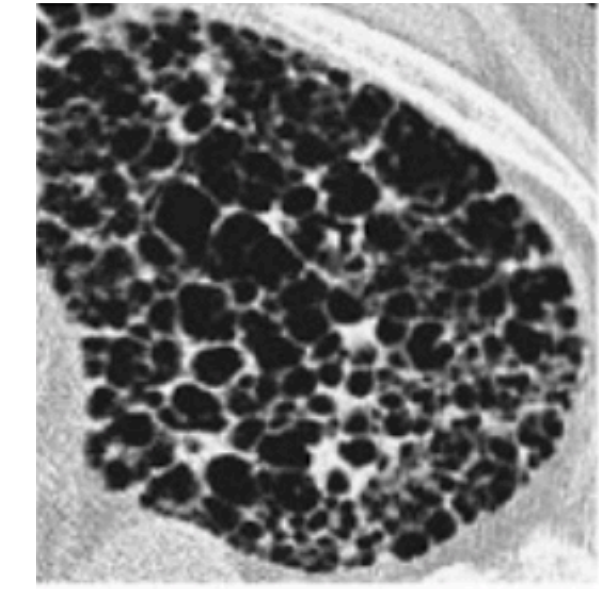
psammomatous



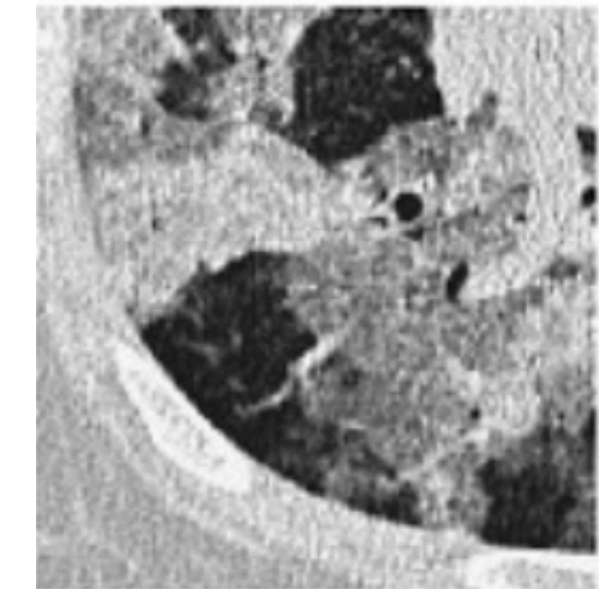
transitional



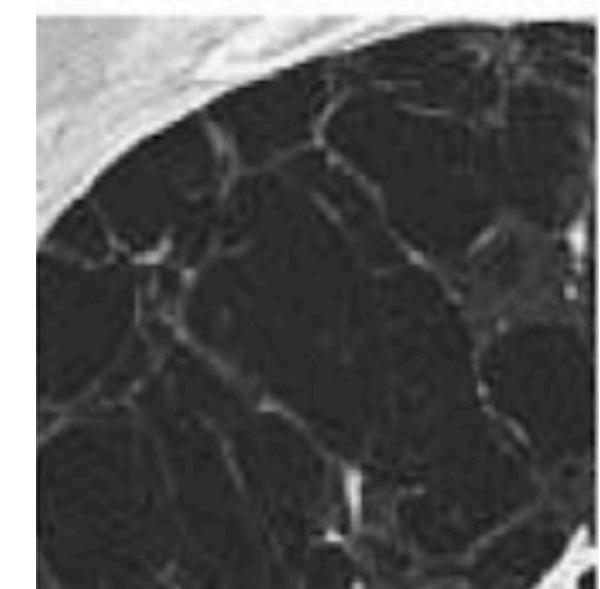
healthy



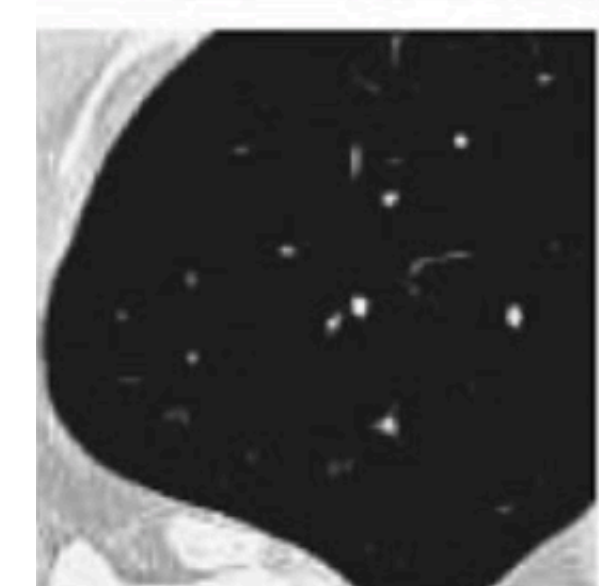
emphysema



ground glass



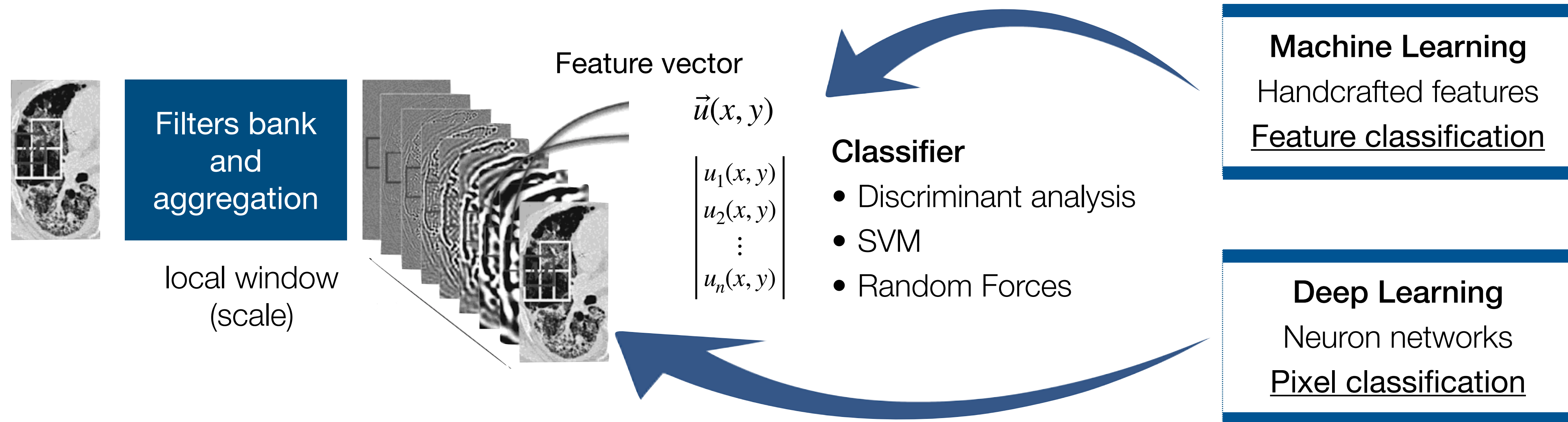
honeycombing



A. Depeursinge, Biomedical Texture Analysis, 2018

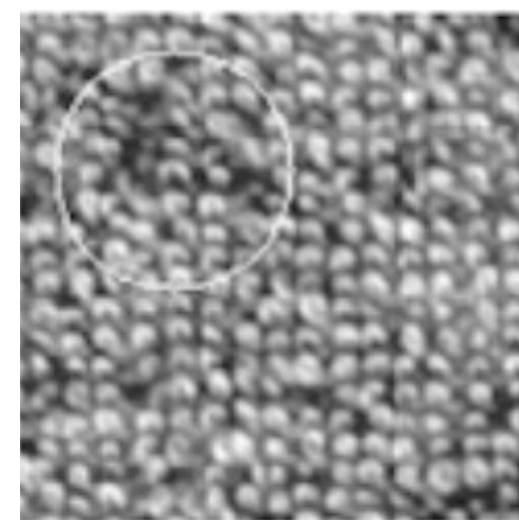


Texture Analysis

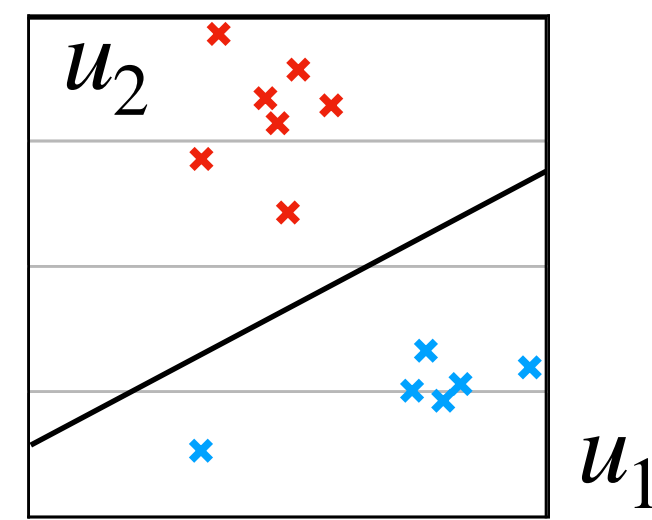
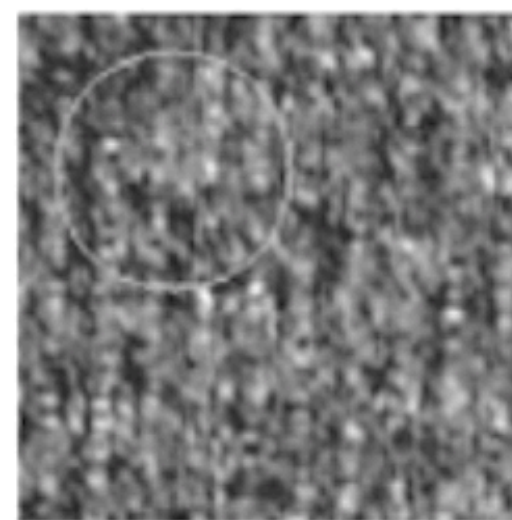


Training

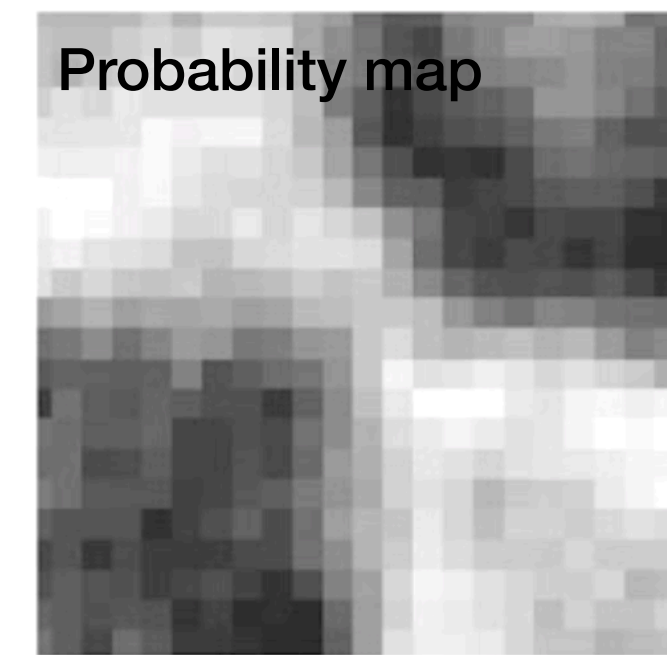
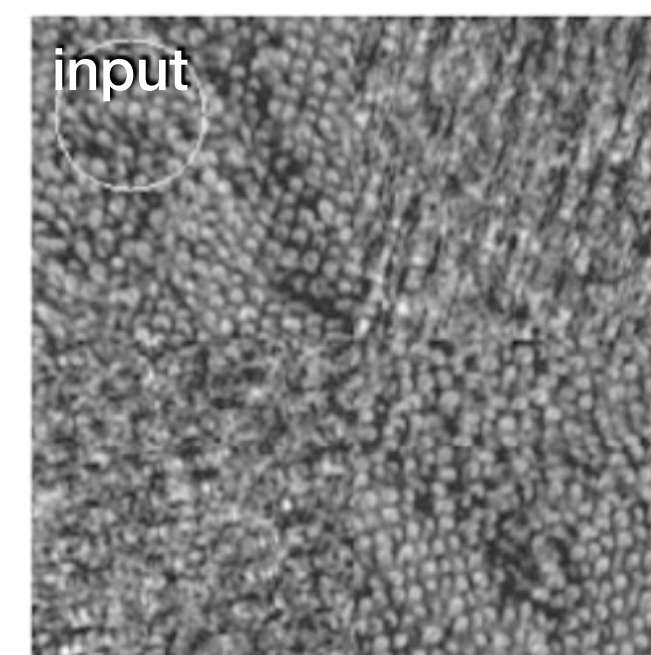
Class1



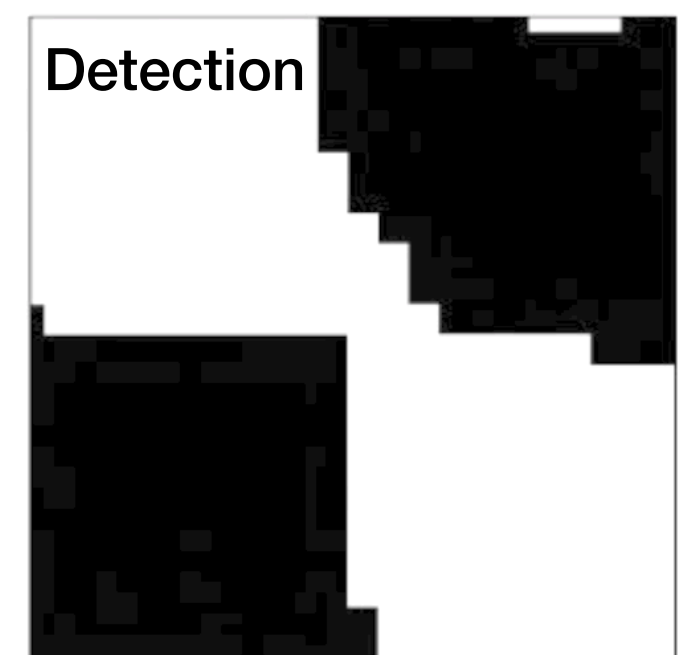
Class2



Predict



$$w_1 u_1 + w_2 u_2$$

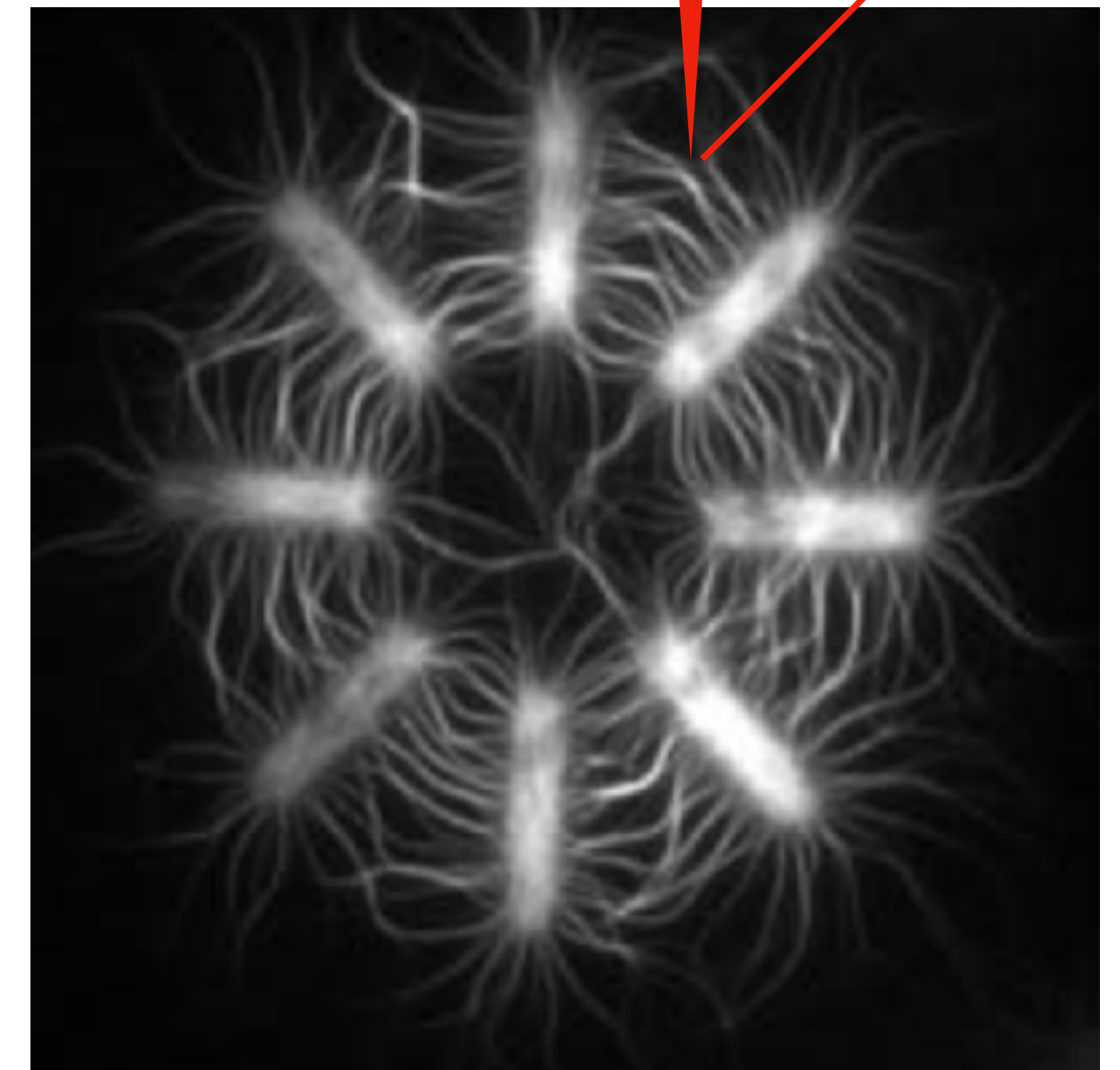
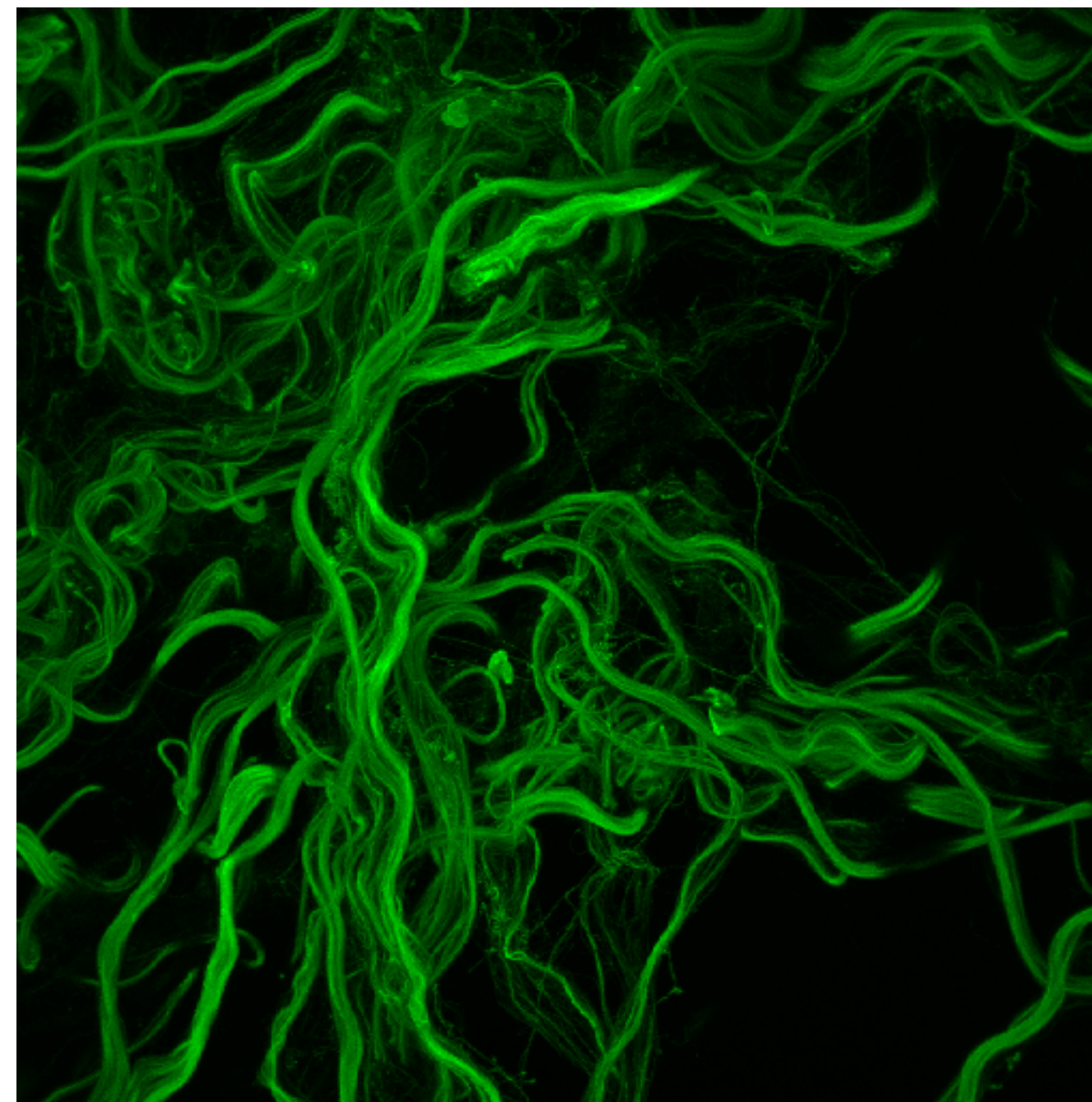
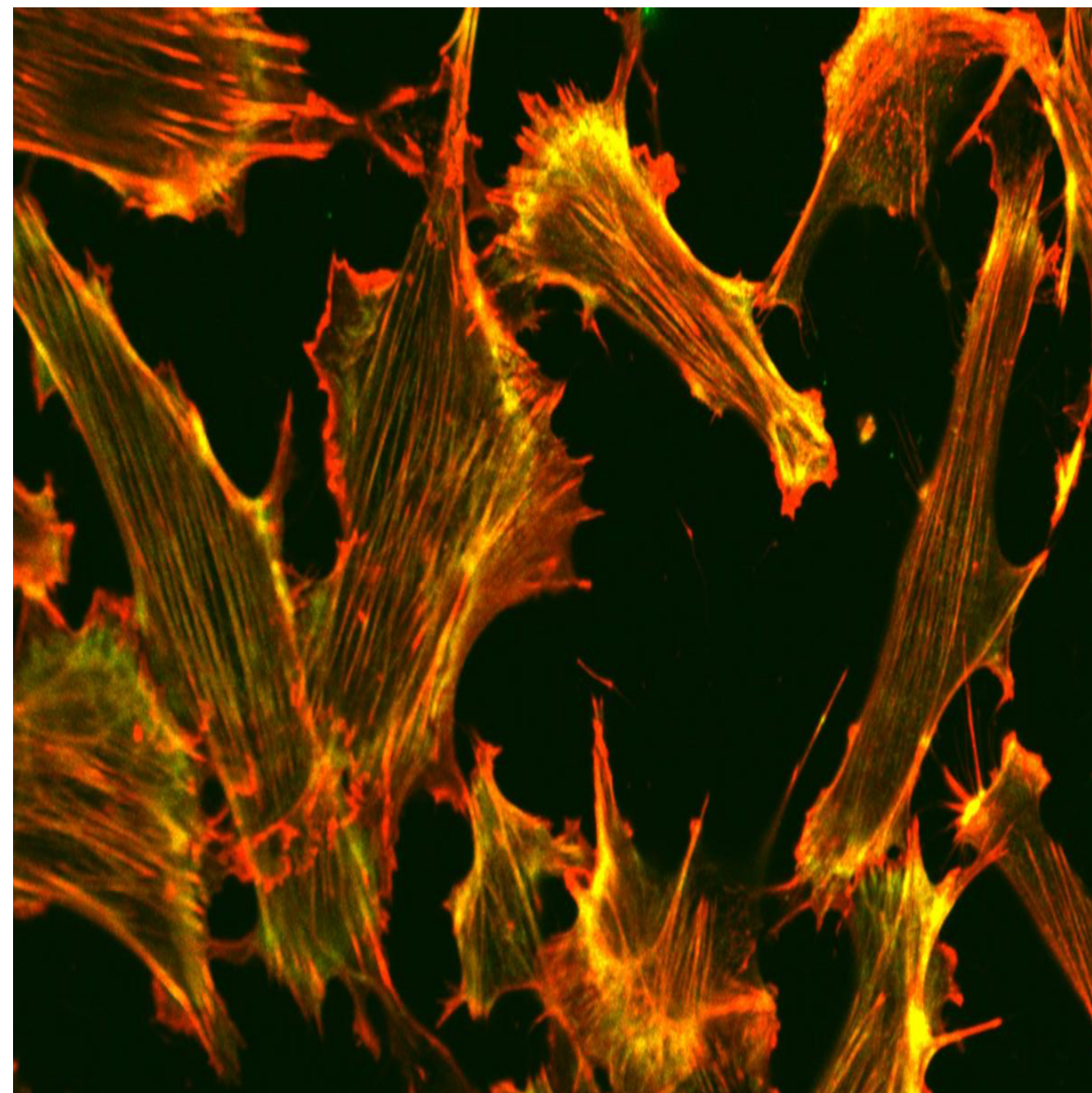




Directional Image Analysis

Direction and orientation information

- Local directional cues: edges, ridges, patterns, texture
- Importance in bioimaging



Direction is
extracted for
derivatives of the
images

David Marr (1980) - Human visual system

"directional clues, edges, ridges, and corners are encoded by a small number of neurons"

Gradient Structure Tensor

Gradient structure tensor at a given position $\mathbf{x}$

$$J(\mathbf{x}) = \int w(\mathbf{y} - \mathbf{x}) (\nabla f(\mathbf{y}) \nabla^T f(\mathbf{y})) d\mathbf{y}$$

Window of observation
non-negative
function
(e.g. Gaussian)

in 2D

$$\nabla f(x, y) \nabla^T f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial y} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \frac{\partial f}{\partial y} \end{bmatrix}$$

Tensor in 2D

$$\begin{bmatrix} \mathcal{S}(g_x^2) & \mathcal{S} g_x g_y \\ \mathcal{S}(g_y g_x) & \mathcal{S}(g_y^2) \end{bmatrix} \rightarrow \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

	λ_1	λ_2
Noisy, no preferred direction	H	H
Tubular-like structure, filament	H	L
Blob-like structure, spot	L	L

Tensor in 3D

$$\begin{bmatrix} \mathcal{S}(g_x^2) & \mathcal{S}(g_x g_y) & \mathcal{S}(g_x g_z) \\ \mathcal{S}(g_y g_x) & \mathcal{S}(g_y^2) & \mathcal{S}(g_y g_z) \\ \mathcal{S}(g_z g_x) & \mathcal{S}(g_z g_y) & \mathcal{S}(g_z^2) \end{bmatrix} \rightarrow \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

	λ_1	λ_2	λ_3
Noisy, no preferred direction	H	H	H
Plate-like structure, membrane	H	H	L
Tubular structure, filament	H	L	L
Blob-like structure, spot	L	L	L

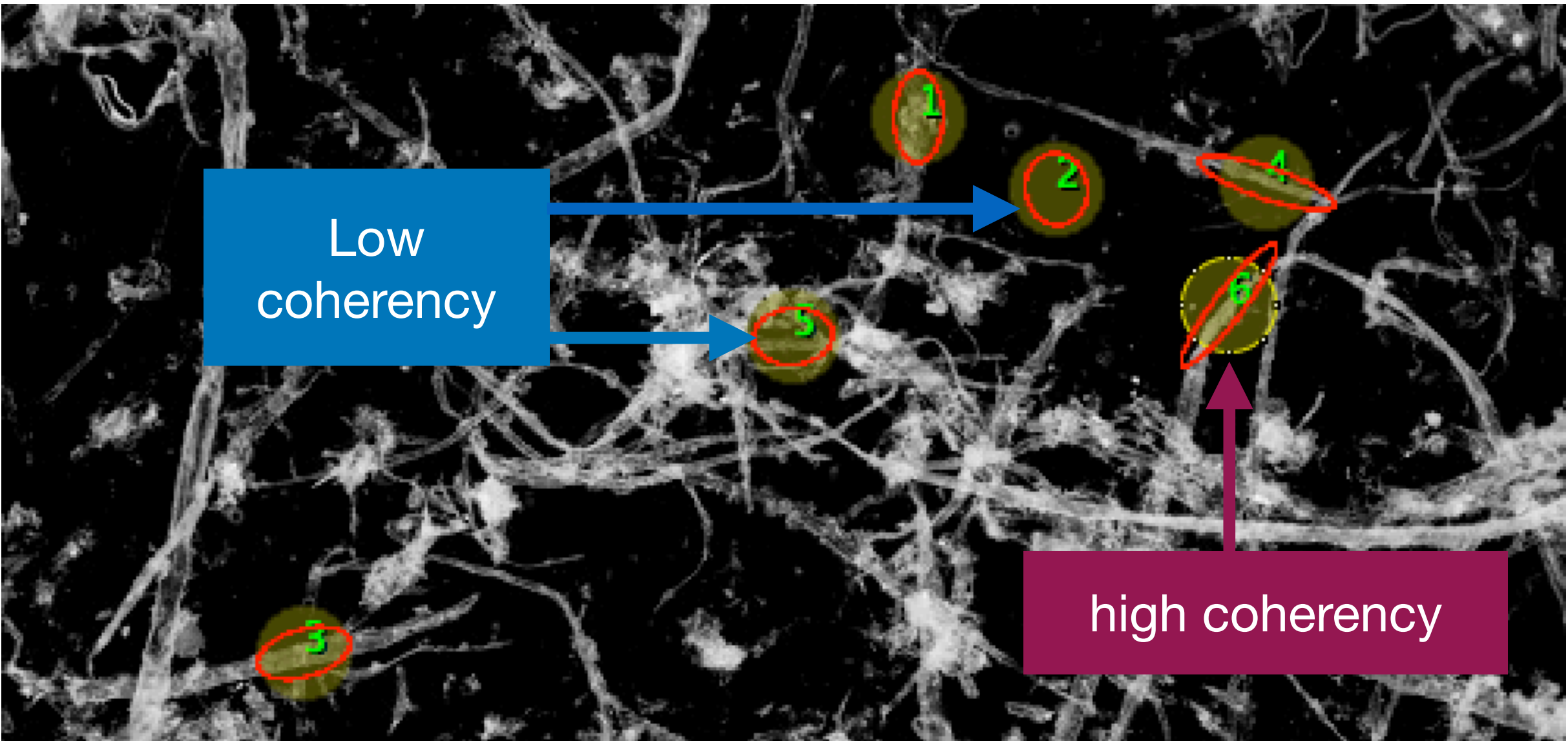
👁 Structure Tensor **Features**

Local features computed from the tensor matrix in 2D

- Gradient energy: $E = \text{trace}(\mathbf{J}) = J_{11} + J_{22}$
- Orientation: $\mathbf{u}_1 = (\cos \theta, \sin \theta)$ with $\theta = \frac{1}{2} \arctan \left(\frac{2J_{12}}{J_{22} - J_{11}} \right)$
- Coherency: $0 \leq C = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} = \frac{\sqrt{(J_{22} - J_{11})^2 + 4J_{12}^2}}{J_{22} + J_{11}} \leq 1$

Tensor Matrix

$$\mathbf{J}(x, y) = \begin{bmatrix} J_{11}(x, y) & J_{12}(x, y) \\ J_{12}(x, y) & J_{22}(x, y) \end{bmatrix}$$



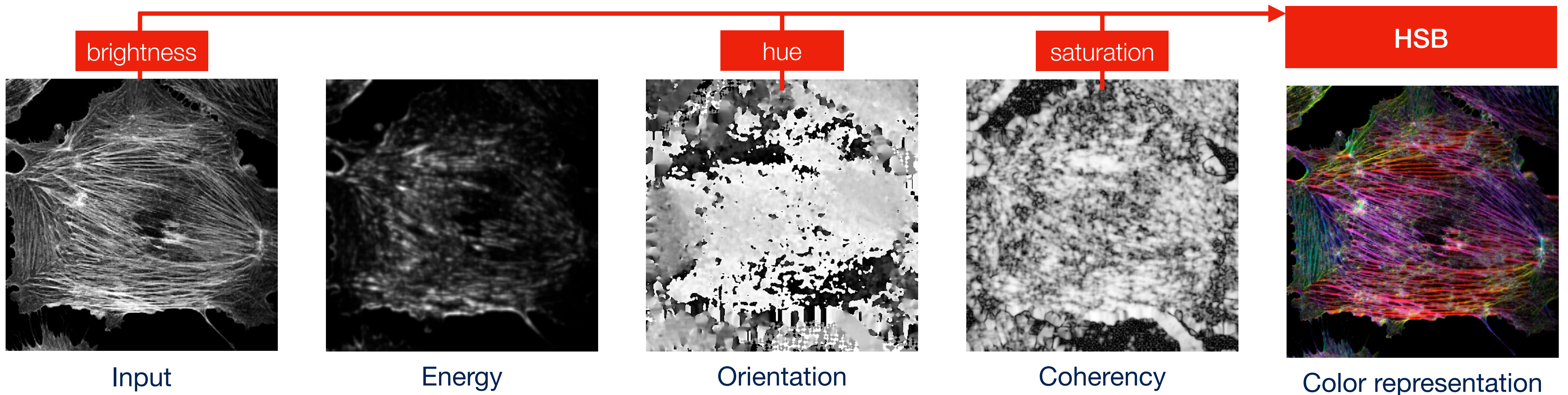
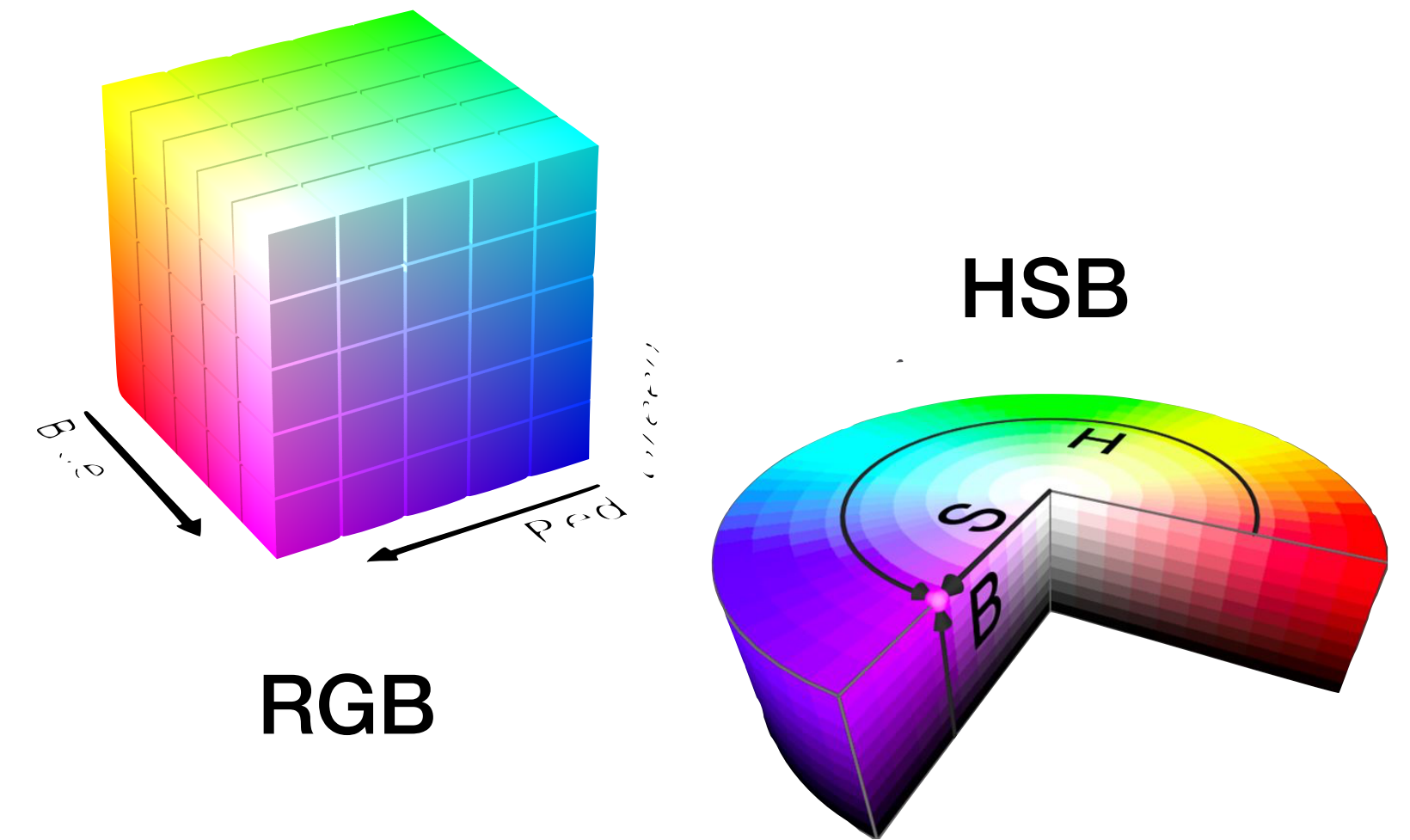
Source: Fiber of paper board, Rosiana Lestiani, Monash University, Australia

	Energy	Orientation	Coherency
1	1294.8	89.04	0.303
2	3.3	-81.86	0.075
3	1082.1	15.67	0.346
4	688.5	-18.46	0.644
5	2606.6	4.60	0.173
6	1695.5	52.74	0.683

👁 Structure Tensor Features

Local features computed from the tensor matrix in 2D

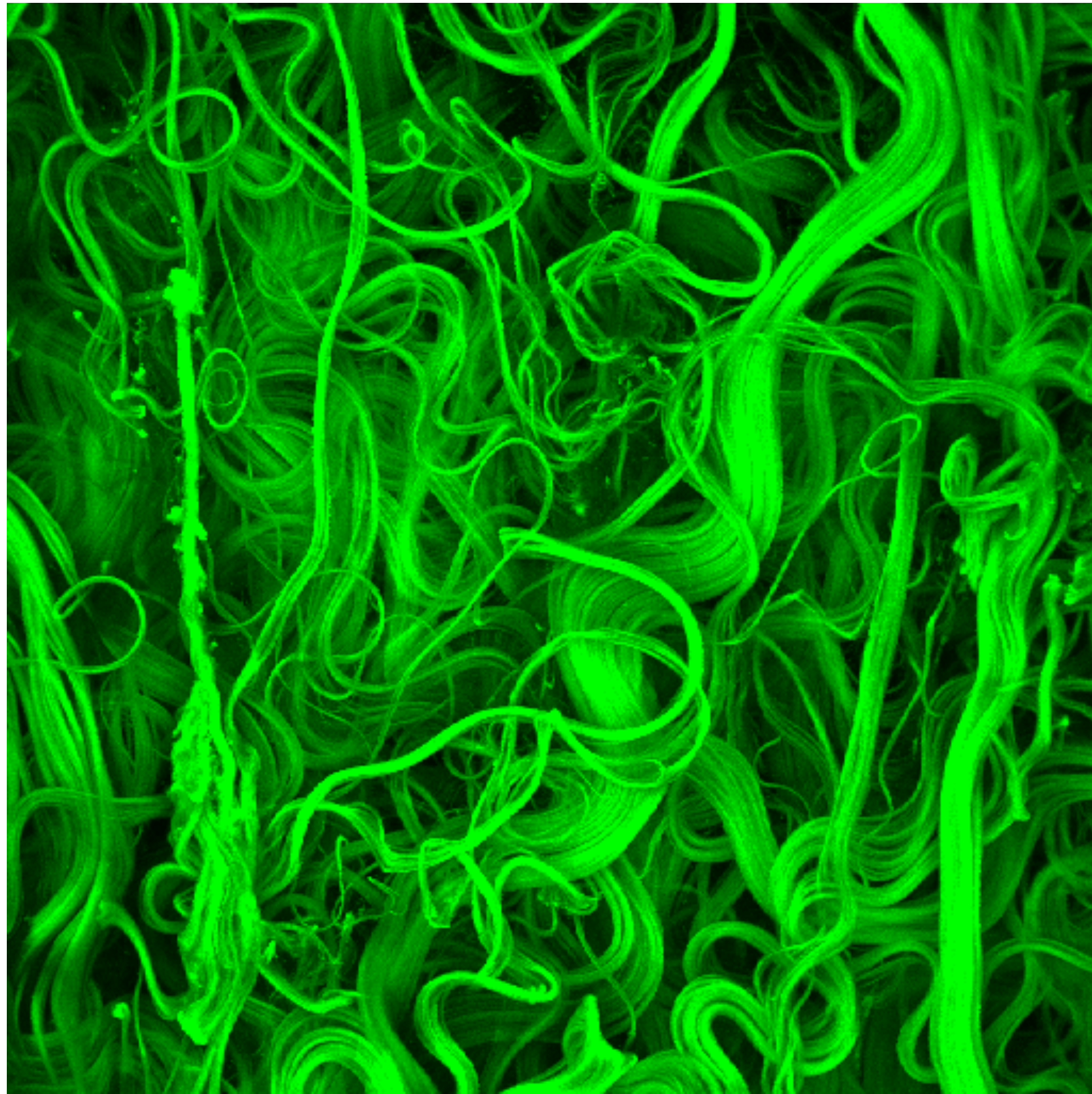
- Gradient energy: $E = \text{trace}(\mathbf{J}) = J_{11} + J_{22}$
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- Coherency: $0 \leq C = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} = \frac{\sqrt{(J_{22} - J_{11})^2 + 4J_{12}^2}}{J_{22} + J_{11}} \leq 1$



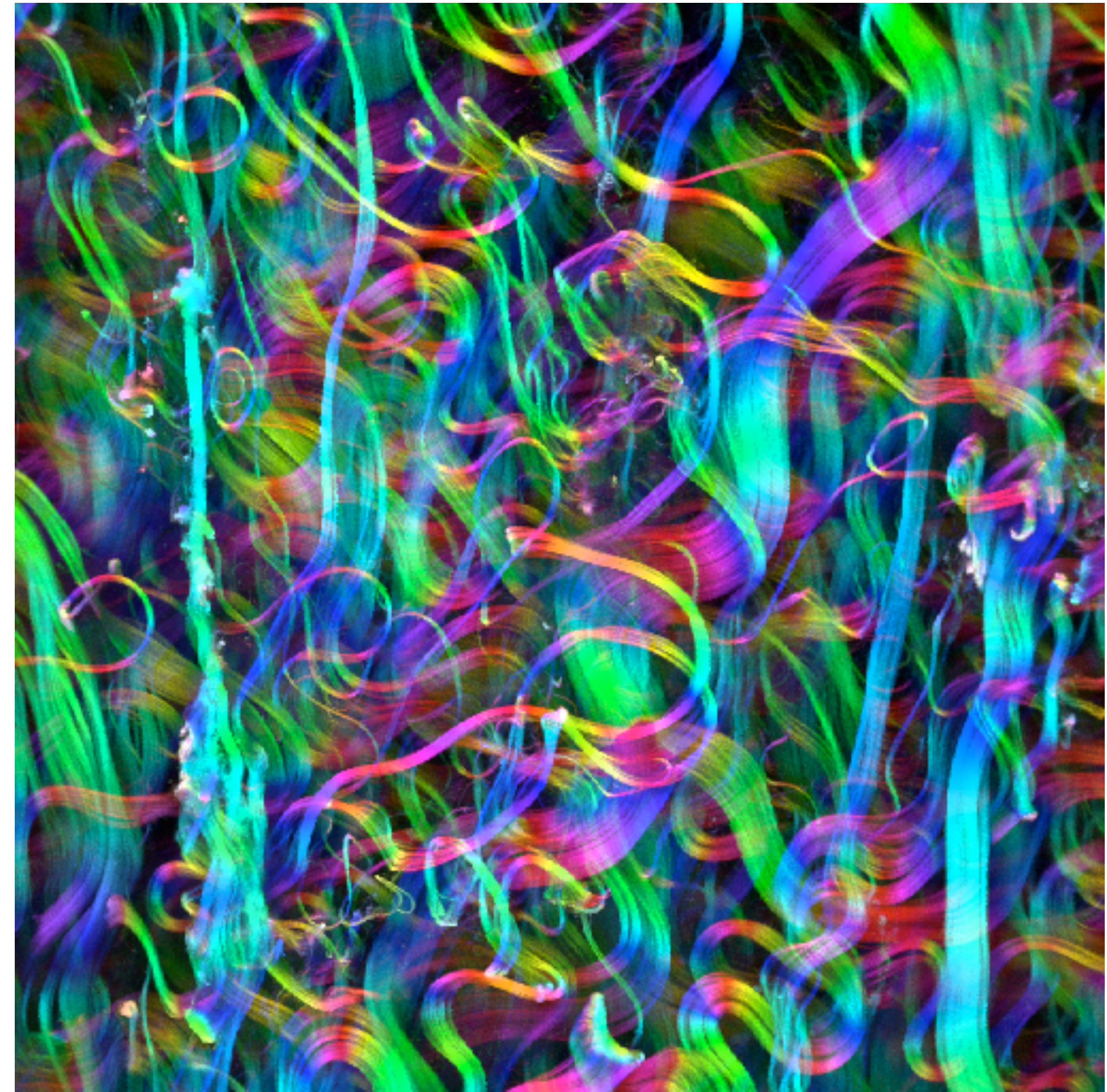


Structure Tensor 3D

Input data (MIP)



Color coded orientation (MIP)



Collagen fiber in adventitia of rabbit carotids ex vivo. 3D confocal microscopy. Rana Rezakhaniha, LHCT, EPFL

Structure Tensor Applications

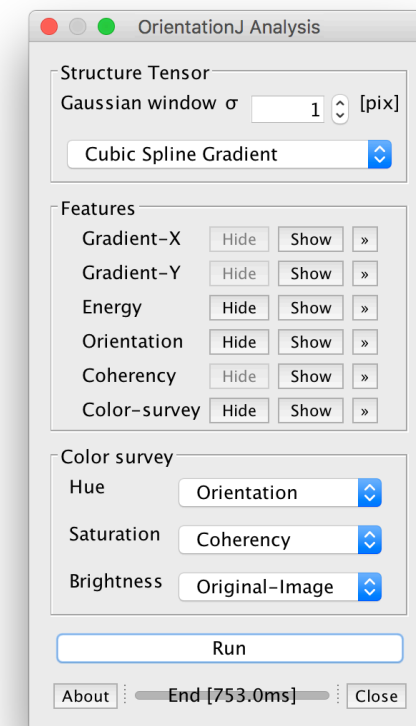
Access to local information

- Based on the gradient
- At the pixel level
- Scale σ
- Orientation
- Coherency

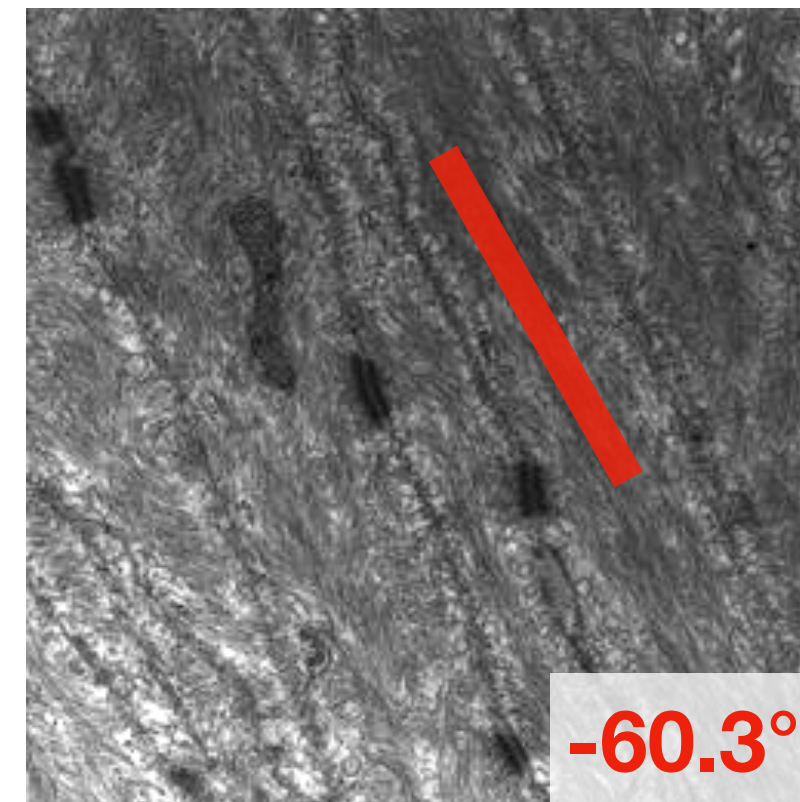


Demonstration

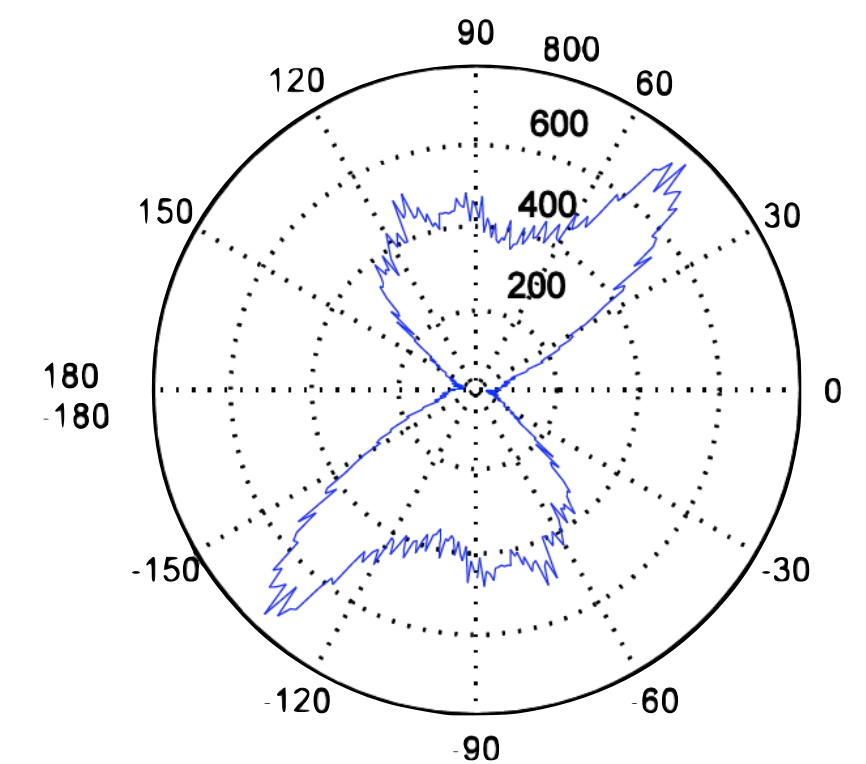
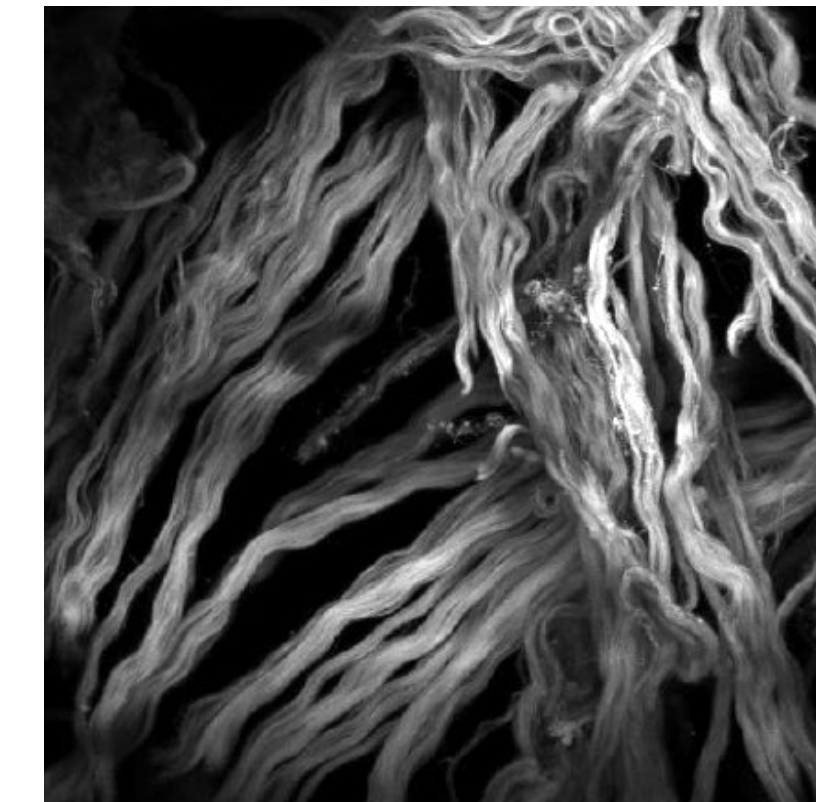
OrientationJ <http://bigwww.epfl.ch/demo/orientation/>



Dominant orientation

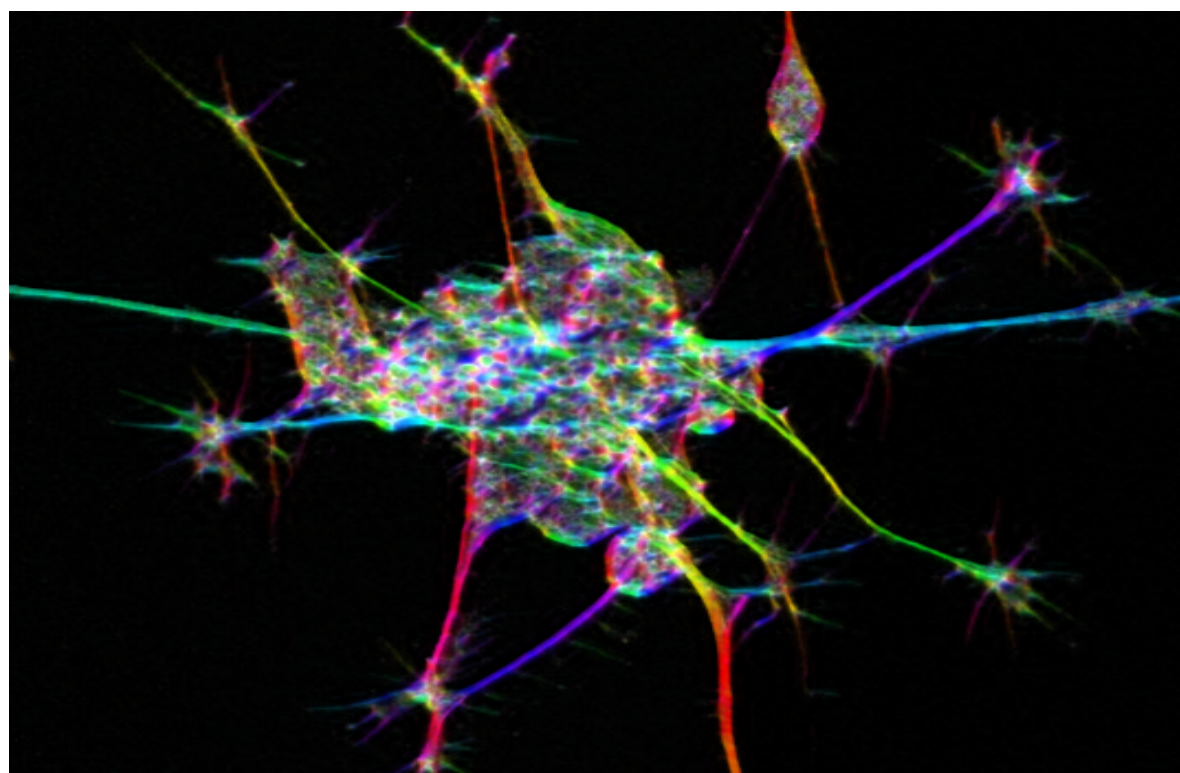


Circular histogram of orientations

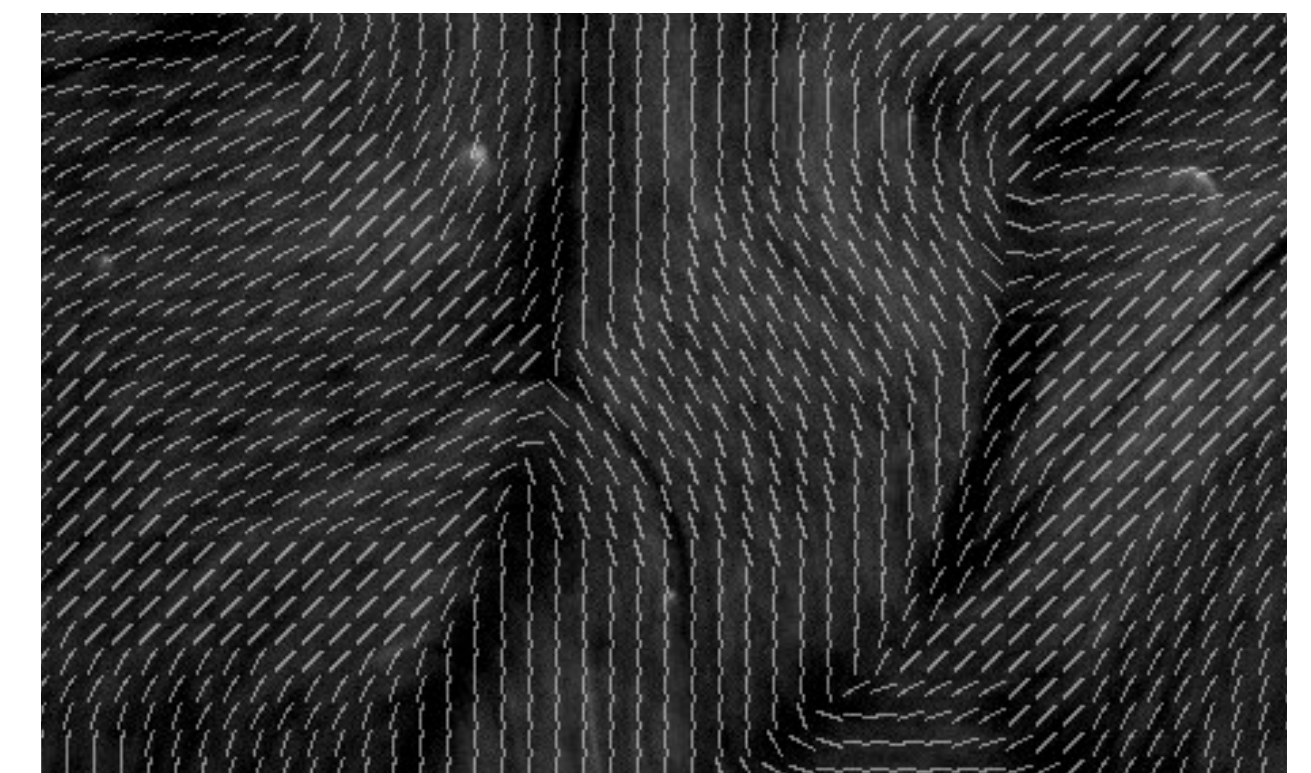
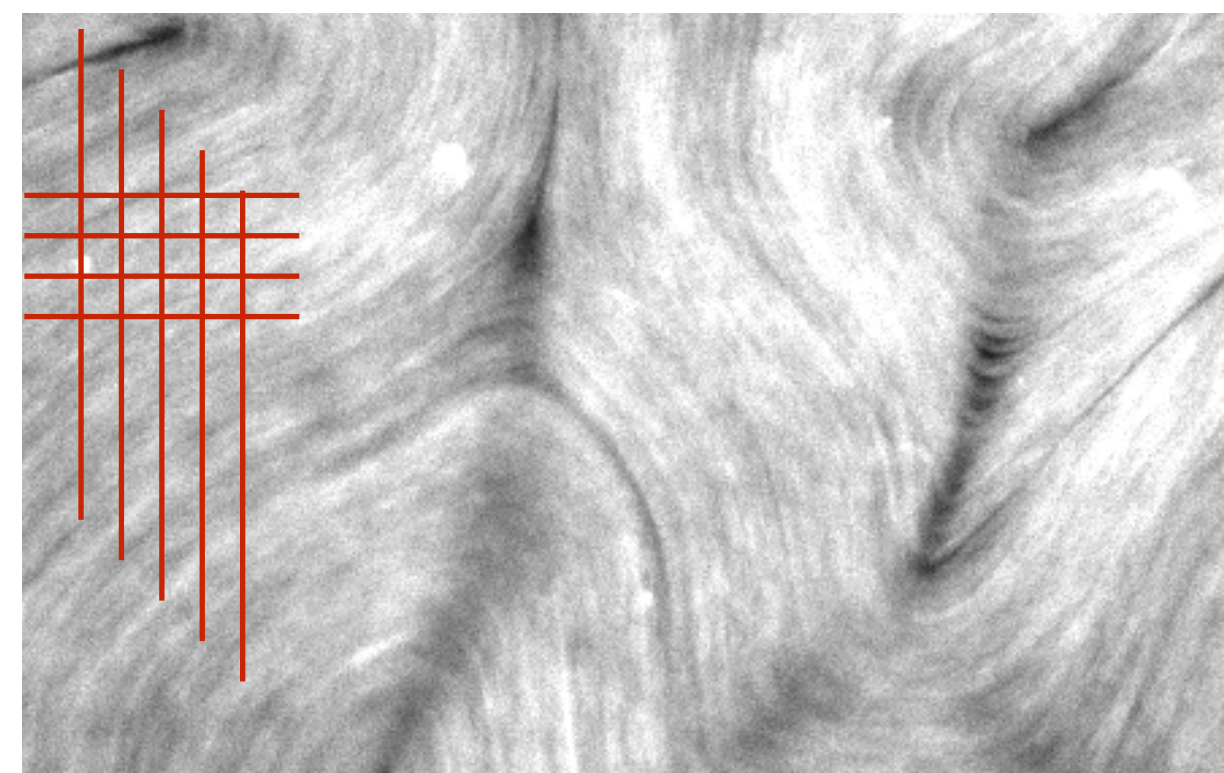


Collagen fiber. Max. intensity projection of confocal images. Rana Rezakhaniha, LHCT, EPFL

Directional analysis



Vector field



Courtesy Jérôme Hardouin